Advanced Analog Building Blocks

Stability & Compensation

Wei SHEN (KIP)

Bode Plot

- real zeros
- zeros with complex conjugates
- real poles
- poles with complex conjugates

http://lpsa.swarthmore.edu/Bode/Bode.html

Term	Magnitude	Phase
Constant: K	$20\log_{10}(\lvert K\rvert)$	K>0: 0° K<0: ±180°
Pole at Origin (Integrator) $\frac{1}{s}$	-20 dB/decade passing through 0 dB at ω=1	-90°
Zero at Origin (Differentiator) s	+20 dB/decade passing through 0 dB at ω=1 *(Mirror image of Integrator about 0 dB)*	+90° *(Mirror image of Integrator about 0°)*
Real Pole $\dfrac{1}{\frac{s}{\omega_0}+1}$	1. Draw low frequency asymptote at 0 dB 2. Draw high frequency asymptote at -20 dB/decade 3. Connect lines at ω_0.	1. Draw low frequency asymptote at 0° 2. Draw high frequency asymptote at -90° 3. Connect with a straight line from $0.1\cdot\omega_0$ to $10\cdot\omega_0$
Real Zero $\frac{s}{\omega_0}+1$	1. Draw low frequency asymptote at 0 dB 2. Draw high frequency asymptote at +20 dB/decade 3. Connect lines at ω_0. *(Mirror image of Real Pole about 0 dB)*	1. Draw low frequency asymptote at 0° 2. Draw high frequency asymptote at +90° 3. Connect with a straight line from $0.1\cdot\omega_0$ to $10\cdot\omega_0$ *(Mirror image of Real Pole about 0°)*
Underdamped Poles (Complex conjugate poles) $\dfrac{1}{\left(\frac{s}{\omega_0}\right)^2+2\zeta\left(\frac{s}{\omega_0}\right)+1},$ $0<\zeta<1$	1. Draw low frequency asymptote at 0 dB 2. Draw high frequency asymptote at -40 dB/decade 3. If ζ<0.5, then draw peak at ω_0 with amplitude $\lvert H(j\omega_0)\rvert=-20\cdot\log_{10}(2\zeta)$, else don't draw peak 4. Connect lines	1. Draw low frequency asymptote at 0° 2. Draw high frequency asymptote at -180° 3. Connect with straight line from $\omega=\frac{\omega_0}{10^{\zeta}}$ to $\omega_0\cdot 10^{\zeta}$ *You can also look in a textbook for examples*
Underdamped Zeros (Complex conjugate zeros) $\left(\frac{s}{\omega_0}\right)^2+2\zeta\left(\frac{s}{\omega_0}\right)+1$ $0<\zeta<1$	1. Draw low frequency asymptote at 0 dB 2. Draw high frequency asymptote at +40 dB/decade 3. If ζ<0.5, then draw peak at ω_0 with amplitude $\lvert H(j\omega_0)\rvert=+20\cdot\log_{10}(2\zeta)$, else don't draw peak 4. Connect lines *(Mirror image of Underdamped Pole about 0 dB)*	1. Draw low frequency asymptote at 0° 2. Draw high frequency asymptote at +180° 3. Connect with straight line from $\omega=\frac{\omega_0}{10^{\zeta}}$ to $\omega_0\cdot 10^{\zeta}$ *You can also look in a textbook for examples.* *(Mirror image of Underdamped Pole about 0°)*

Bode Plot – example

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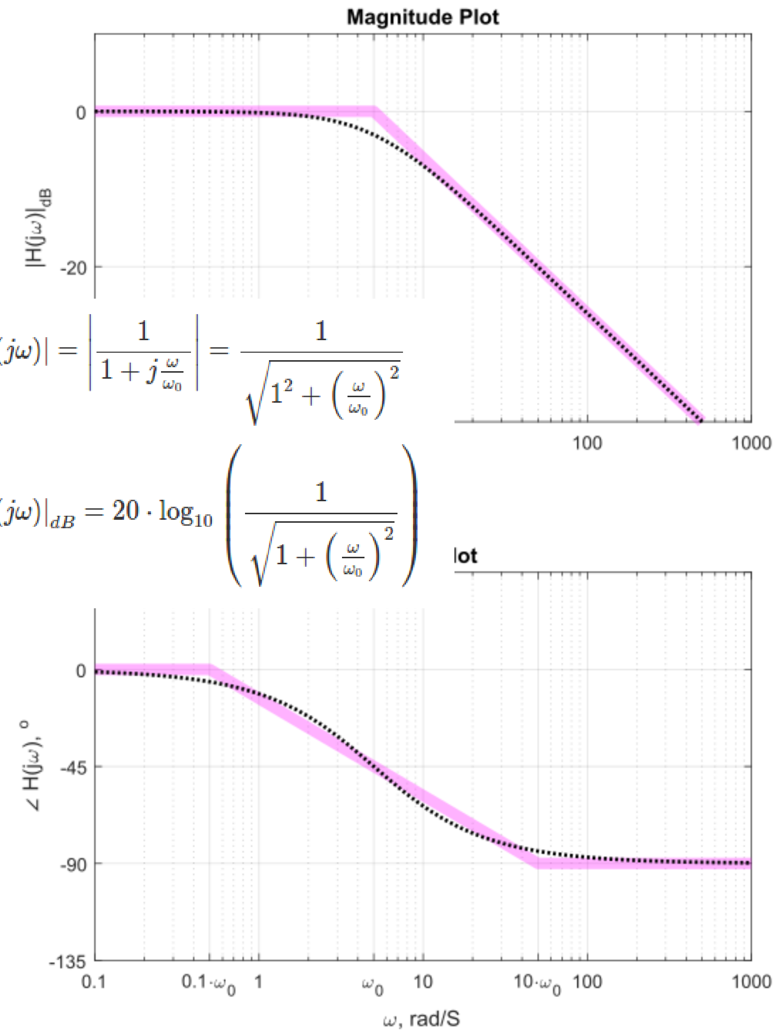


$$|H(j\omega)| = \left| \frac{1}{1 + j\frac{\omega}{\omega_0}} \right| = \frac{1}{\sqrt{1^2 + \left(\frac{\omega}{\omega_0}\right)^2}}$$

$$|H(j\omega)|_{dB} = 20 \cdot \log_{10} \left(\frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^2}} \right)$$

Real Poles :

$$\frac{1}{\frac{s}{\omega_0} + 1}$$

Amplitude

- Draw low frequency asymptote at 0 dB
- Draw high frequency asymptote at -20 dB/decade
- Connect lines at ω_0.

Phase

- Draw low frequency asymptote at 0°
- Draw high frequency asymptote at -90°
- Connect with a straight line from $0.1 \cdot \omega_0$ to $10 \cdot \omega_0$

Second Order Real Poles :

$$\frac{1}{\left(\frac{s}{\omega_0} + 1\right)^2}$$

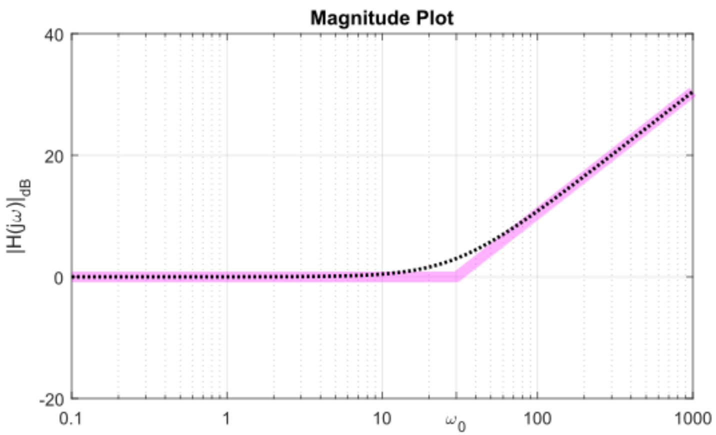
Amplitude

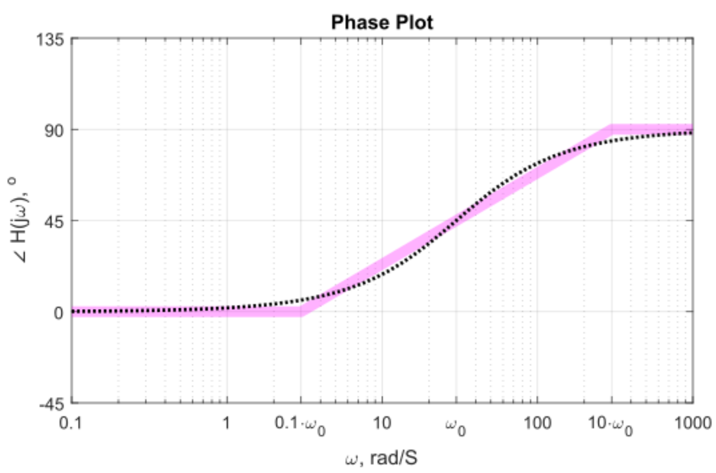
- Draw low frequency asymptote at 0 dB
- Draw high frequency asymptote at -40 dB/decade
- Connect lines at break frequency.

Phase

- Draw low frequency asymptote at 0°
- Draw high frequency asymptote at -180°
- Connect with a straight line from $0.1 \cdot \omega_0$ to $10 \cdot \omega_0$

Bode Plot – example





Real Negative Zeros :

$$\frac{s}{\omega_0} + 1$$

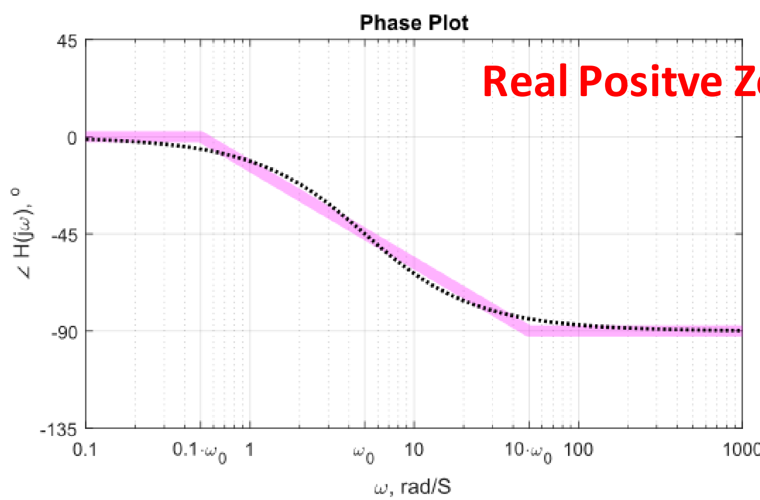
Amplitude

- Draw low frequency asymptote at 0 dB
- Draw high frequency asymptote at +20 dB/decade
- Connect lines at ω_0.

Phase

- Draw low frequency asymptote at 0°
- Draw high frequency asymptote at +90°
- Connect with a straight line from $0.1 \cdot \omega_0$ to $10 \cdot \omega_0$

Real Positve Zeros :



Bode Plot – example

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$$H(s) = \frac{\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2} = \frac{1}{\left(\frac{s}{\omega_0}\right)^2 + 2\zeta\left(\frac{s}{\omega_0}\right) + 1}$$

Complex Conjugate Poles :

Amplitude:

$$|H(j\omega)| = \left|\frac{1}{\left(\frac{j\omega}{\omega_0}\right)^2 + 2\zeta\left(\frac{j\omega}{\omega_0}\right) + 1}\right| = \left|\frac{1}{-\left(\frac{\omega}{\omega_0}\right)^2 + j2\zeta\left(\frac{\omega}{\omega_0}\right) + 1}\right| = \left|\frac{1}{\left(1 - \left(\frac{\omega}{\omega_0}\right)^2\right) + j\left(2\zeta\left(\frac{\omega}{\omega_0}\right)\right)}\right|$$

$$= \frac{1}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_0}\right)^2\right)^2 + \left(2\zeta\frac{\omega}{\omega_0}\right)^2}}$$

$$|H(j\omega)|_{dB} = -20 \cdot \log_{10}\left(\sqrt{\left(1 - \left(\frac{\omega}{\omega_0}\right)^2\right)^2 + \left(2\zeta\frac{\omega}{\omega_0}\right)^2}\right)$$

Phase:

$$\angle H(j\omega) = \angle\left(\frac{1}{\left(\frac{j\omega}{\omega_0}\right)^2 + 2\zeta\left(\frac{j\omega}{\omega_0}\right) + 1}\right) = -\angle\left(\left(\frac{j\omega}{\omega_0}\right)^2 + 2\zeta\left(\frac{j\omega}{\omega_0}\right) + 1\right) = -\angle\left(1 - \left(\frac{\omega}{\omega_0}\right)^2 + 2\zeta\left(\frac{j\omega}{\omega_0}\right)\right)$$

$$= -\arctan\left(\frac{2\zeta\frac{\omega}{\omega_0}}{1 - \left(\frac{\omega}{\omega_0}\right)^2}\right)$$

Bode Plot – example

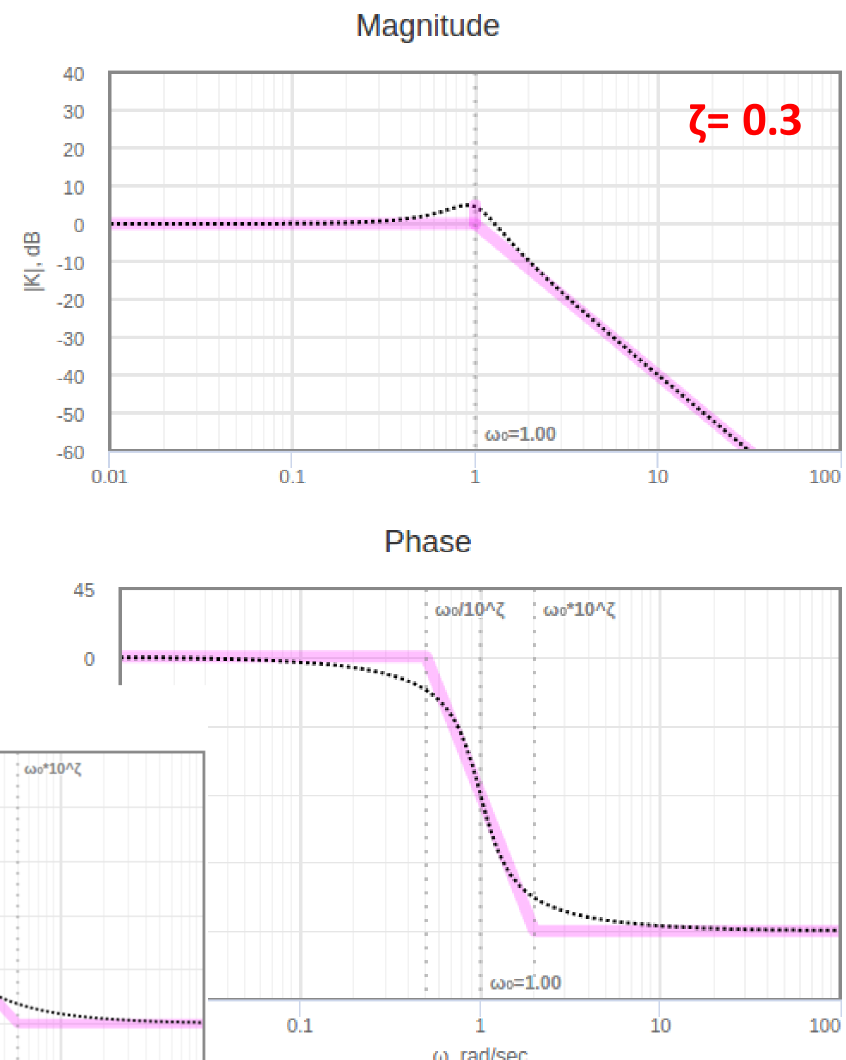
Complex Conjugate Poles :

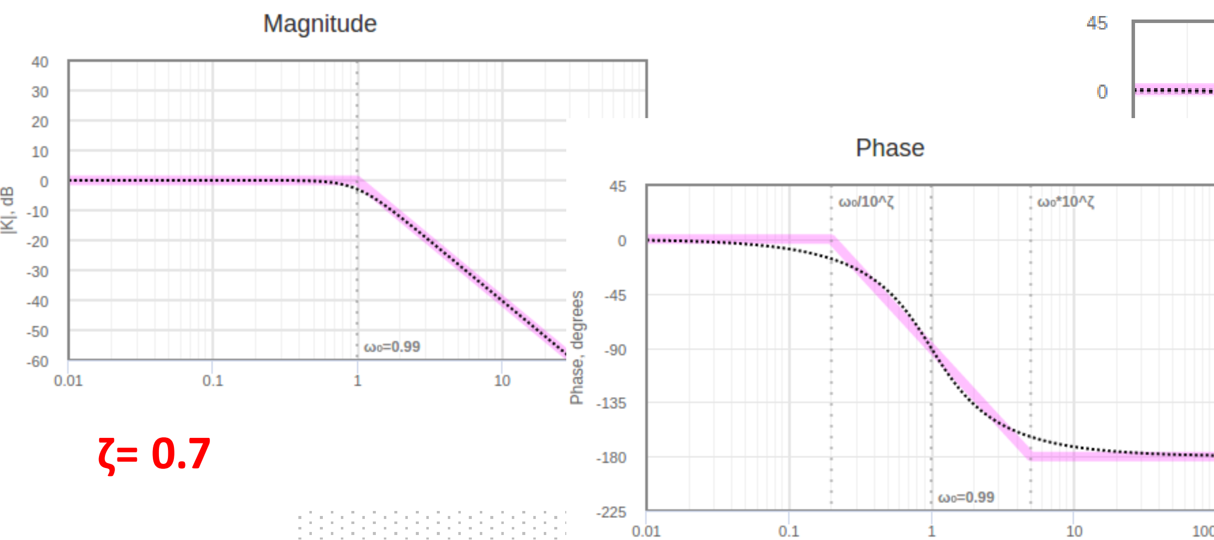
Amplitude

- Draw low frequency asymptote at 0 dB
- Draw high frequency asymptote at -40 dB/decade
- If $\zeta<0.5$, then draw peak at ω0 with amplitude $|H(j\omega_0)|=-20\cdot\log10(2\zeta)$, else don't draw peak
- Connect lines

Phase

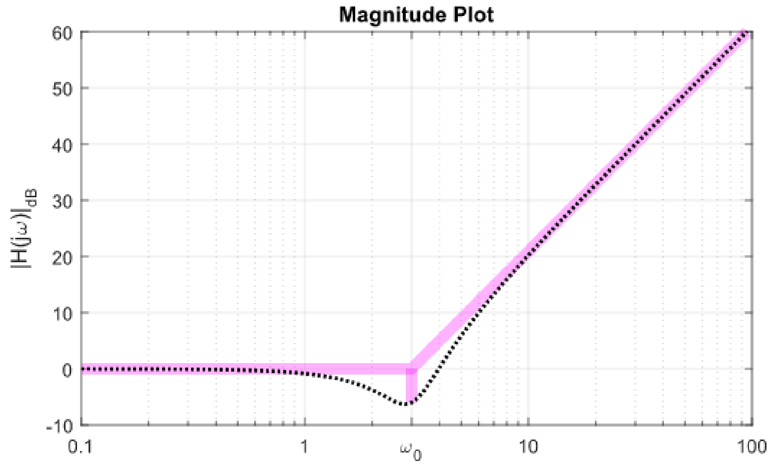
- Draw low frequency asymptote at 0°
- Draw high frequency asymptote at -180°
- Connect with straight line from $\omega_0 / 10^{\zeta}$ to $\omega_0 * 10^{\zeta}$

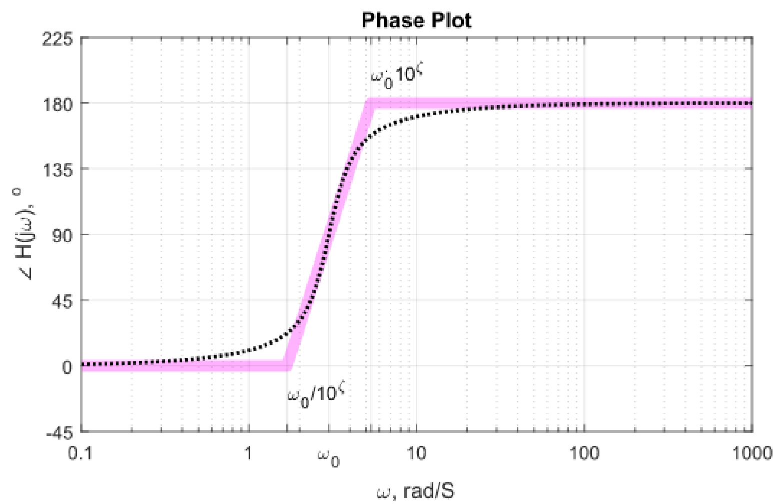




ζ= 0.7

Bode Plot – example





Complex Conjugate Zeros :

Amplitude

- Draw low frequency asymptote at 0 dB
- Draw high frequency asymptote at +40 dB/decade
- If ζ<0.5, then draw peak at ω0 with amplitude |H(jω0)|=+20·log10(2ζ), else don't draw peak
- Connect lines

Phase

- Draw low frequency asymptote at 0°
- Draw high frequency asymptote at +180°
- Connect with straight line from $\omega_0 / 10^{\zeta}$ to $\omega_0 * 10^{\zeta}$

Bode Plot – example

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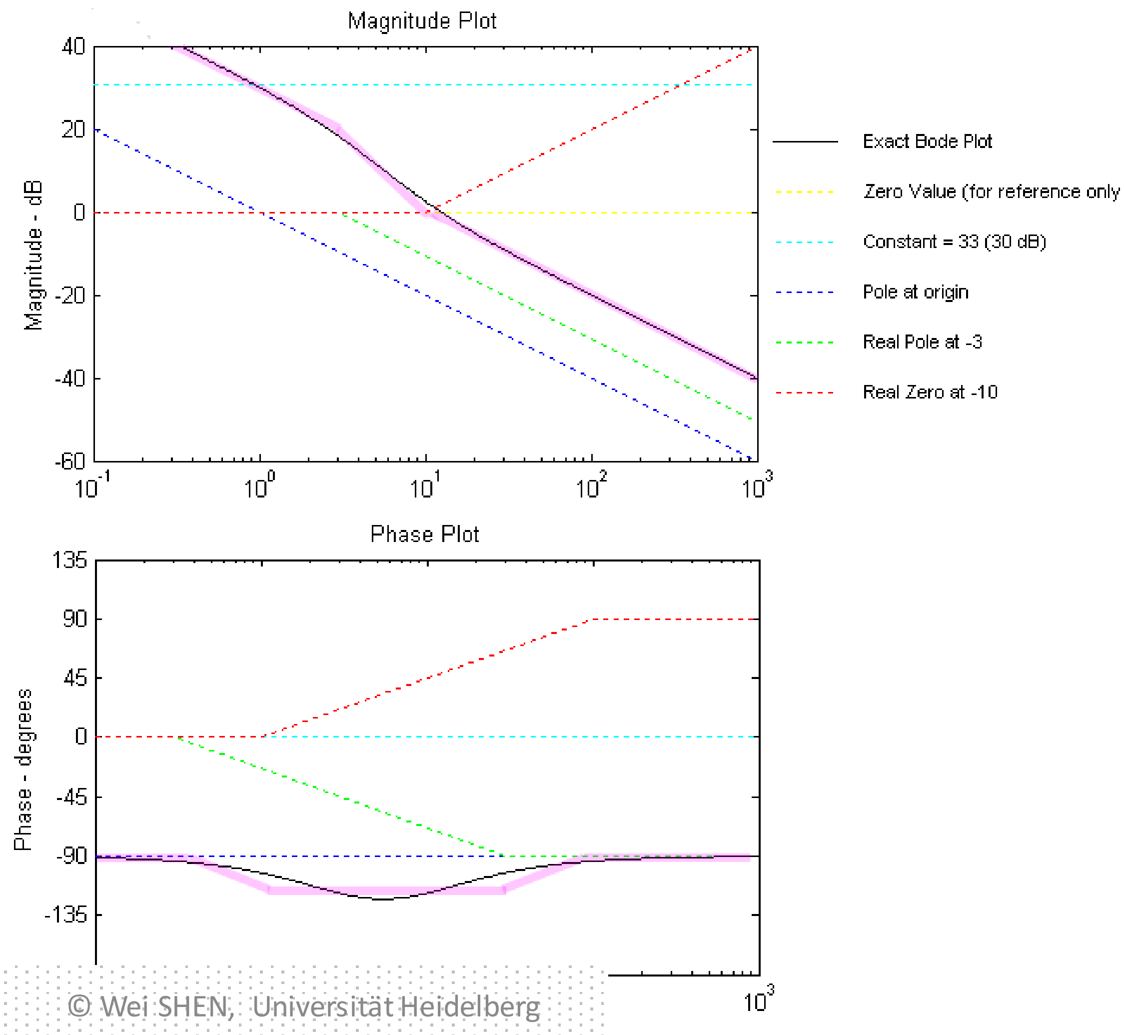
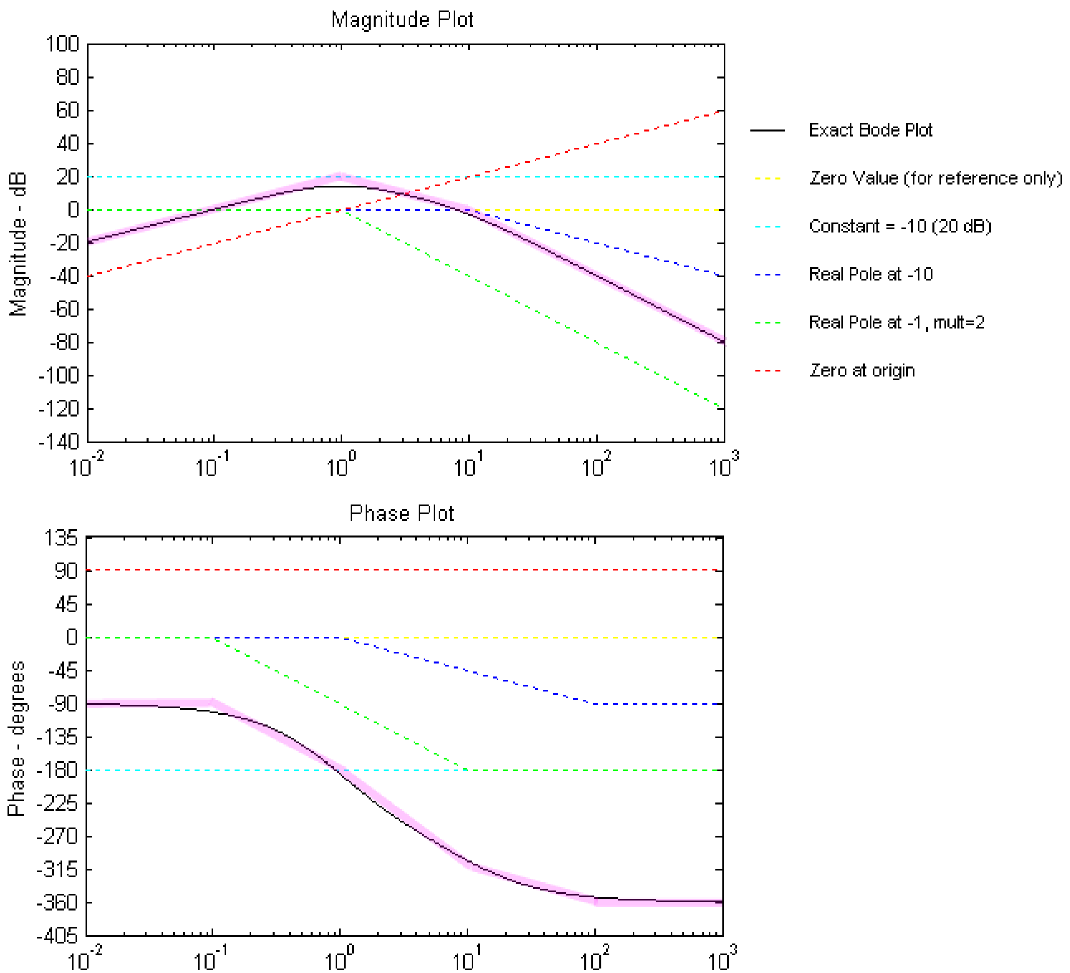
Asymptotic Bode Plot

$$H(s) = 10\frac{s+10}{s^2+3s}$$



Bode Plot – example

Asymptotic Bode Plot $H(s) = -100\frac{s}{s^3 + 12s^2 + 21s + 10}$

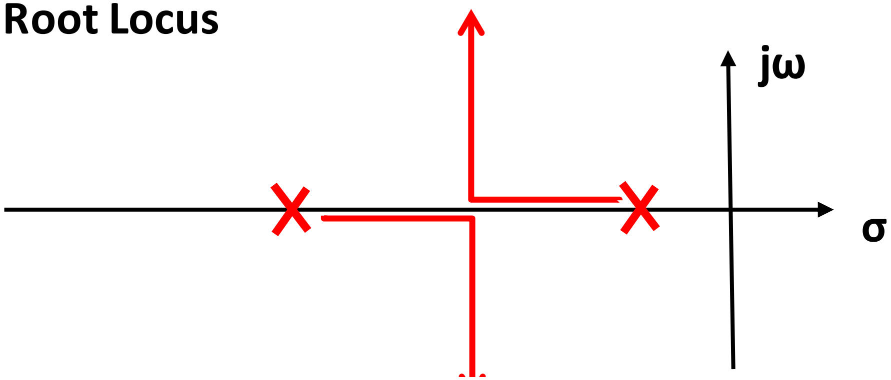


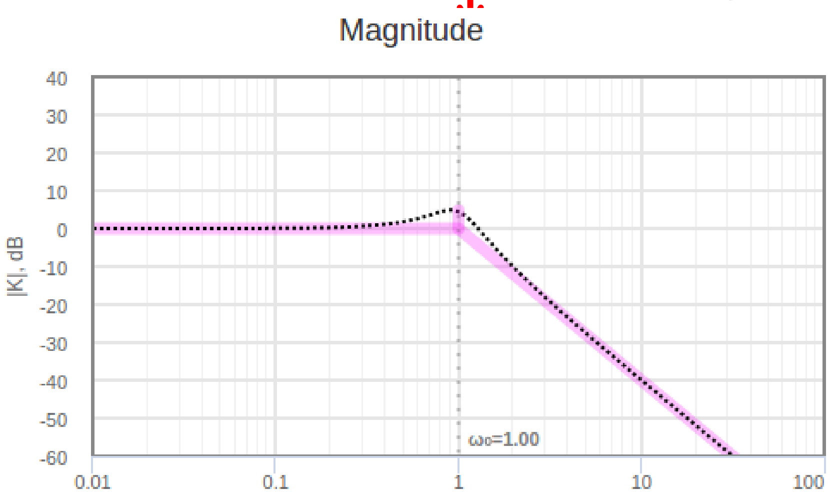
Stability of 2 Pole System

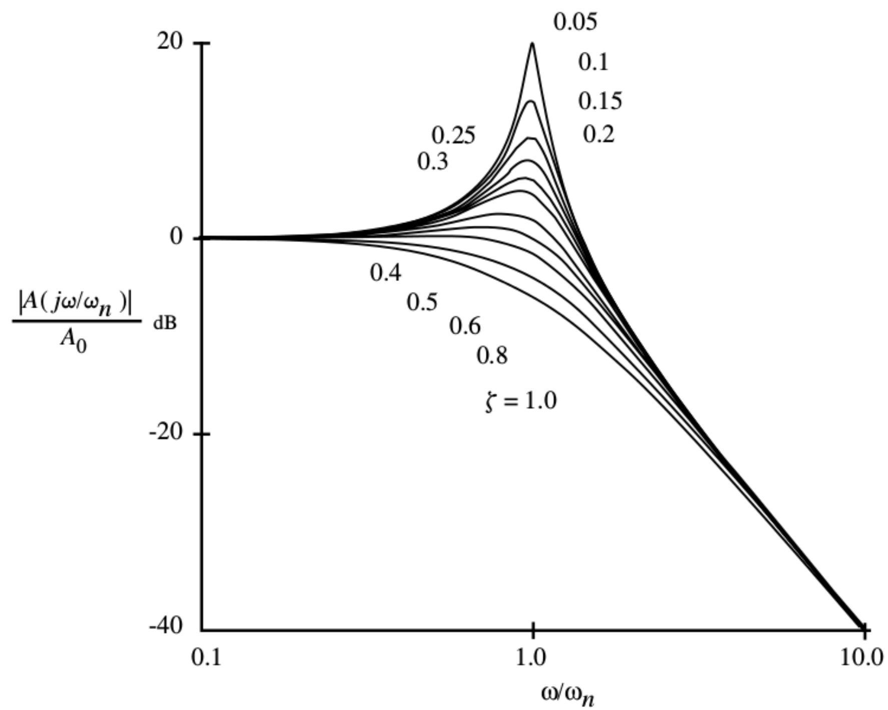
$$H(s) = \frac{A_0\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2} = \frac{A_0}{\left(\frac{s}{\omega_0}\right)^2 + 2\zeta\left(\frac{s}{\omega_0}\right) + 1}$$

Transfer Function : $\frac{A_{out}}{A_{in}}$

Root Locus







Stability of 2 Pole System

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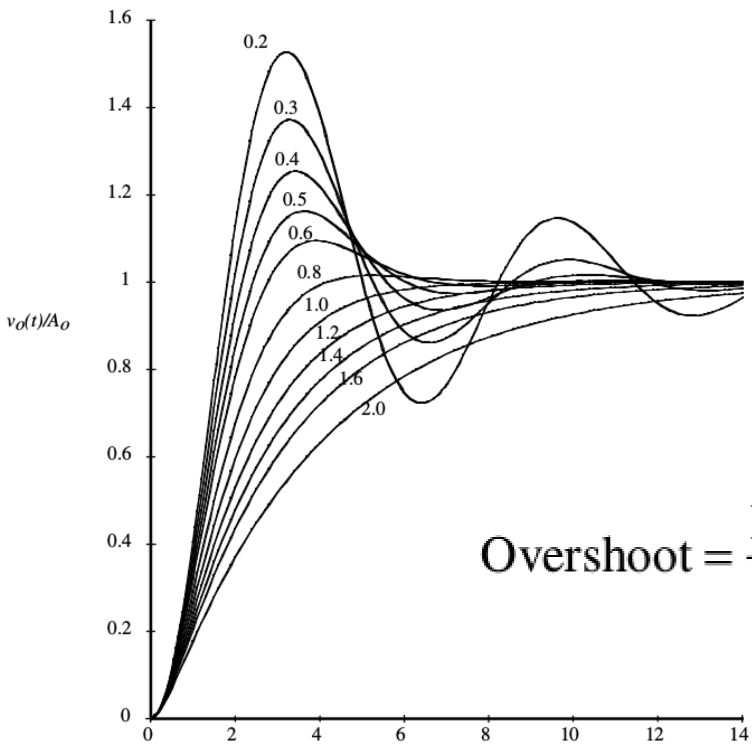
- **Unit Step Response** in the time domain

how to calculate?

$$v_o(t) = A_0\left[1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t}\sin\left(\sqrt{1-\zeta^2}\,\omega_n t + \phi\right)\right]$$

with

$$\phi = \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)$$



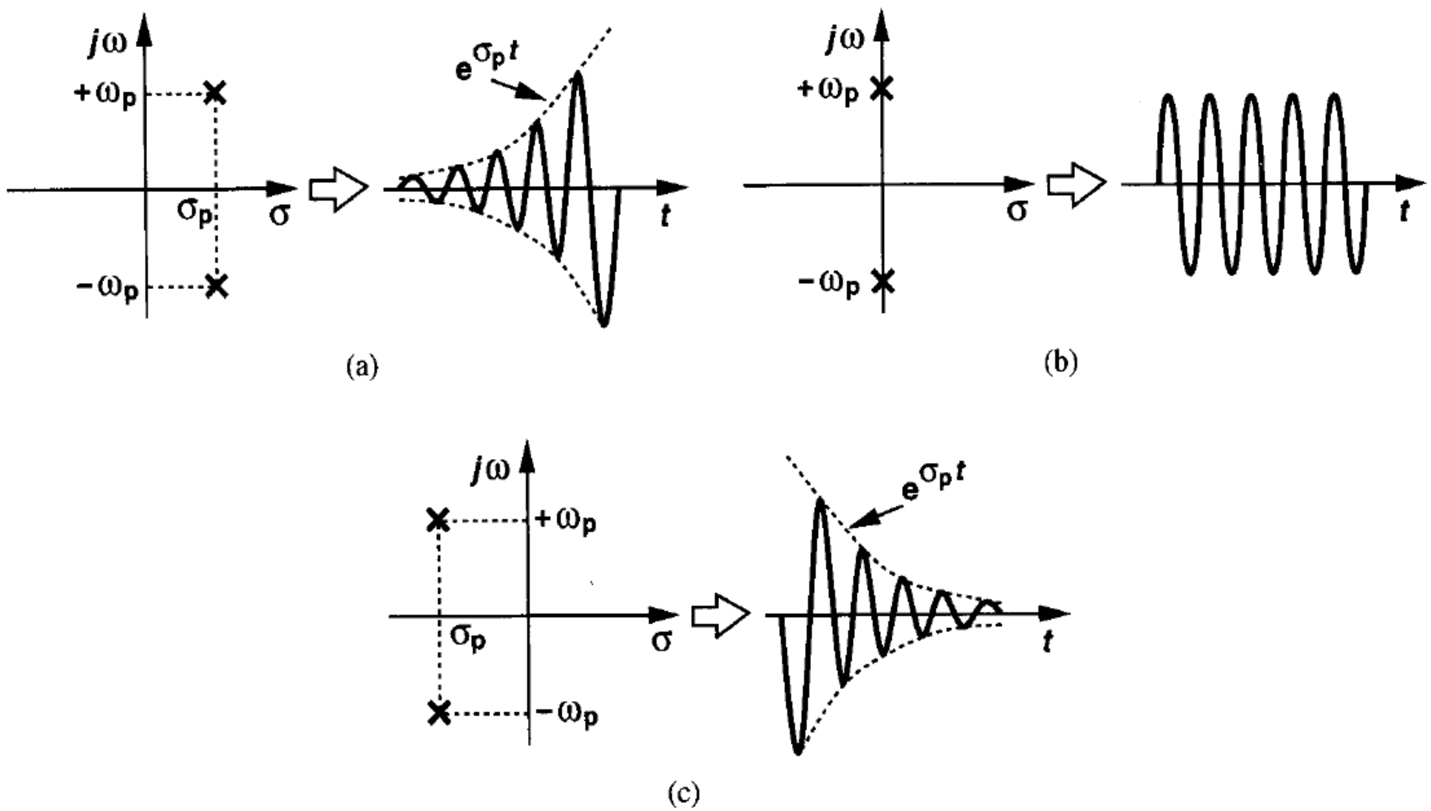
As $\zeta \rightarrow 0$, the unit step response oscillate
Usually, ζ is set to 0.7 – 0.8 to enhance response speed

$$\text{Overshoot} = \frac{\text{Peak value} - \text{Final value}}{\text{Final value}} = \exp\left(\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right)$$

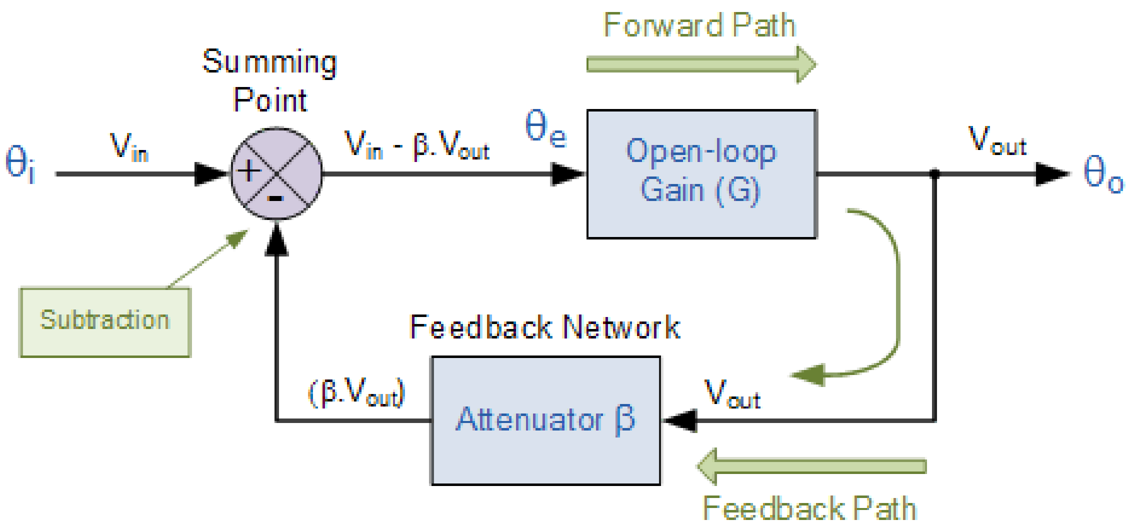
What about a impulse response ??????

Stability of 2 Pole System

What about a impulse response ??????



Stability of Feedback Systems



$$\frac{Y}{X}(s) = \frac{H(s)}{1 + \beta H(s)},$$

β <= 1

If $\beta H(s) = -1$, then the transfer function goes to infinity which means :

@ certain frequency, $|\beta H(j\omega_0)| = 1$ & $\angle \beta H(j\omega_0) = -180^\circ$

In total, 360° phase shift, as negative feedback is used

Oscillation builds up with 360° feedback and positive amplitude

the amplitude feedbacked should be more than unity

Stability of Feedback Systems

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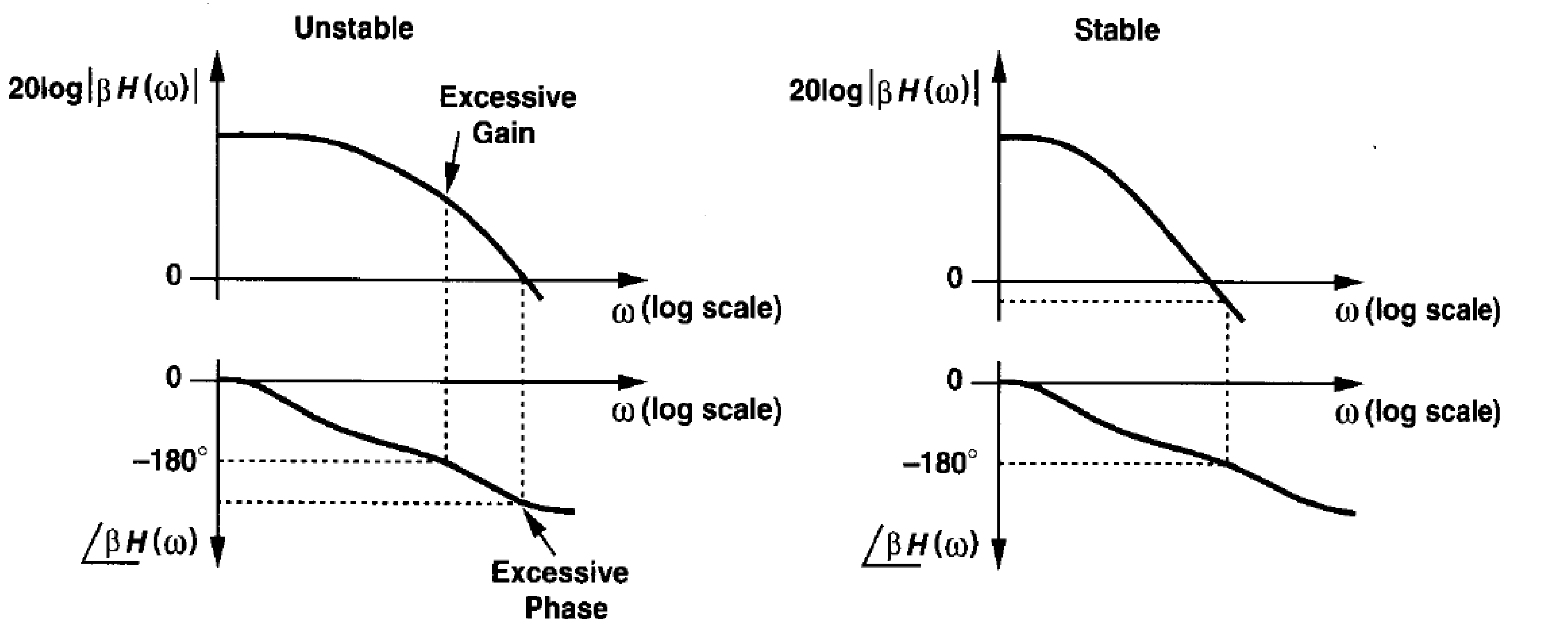
$$\frac{Y}{X}(s) = \frac{H(s)}{1+\beta H(s)},$$

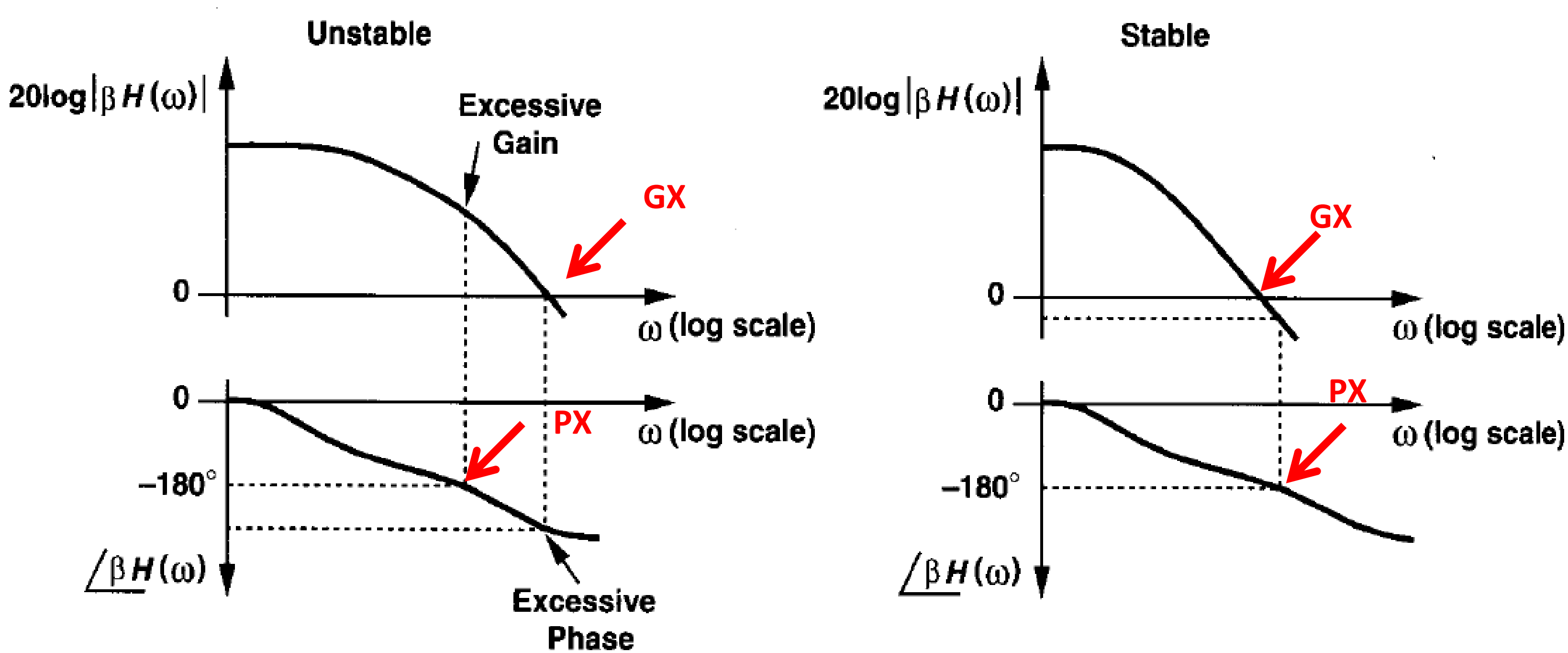
the frequency response of βH(s) is always used to indicate the stability of the system

by knowing the βH(s) , already able to tell the stability



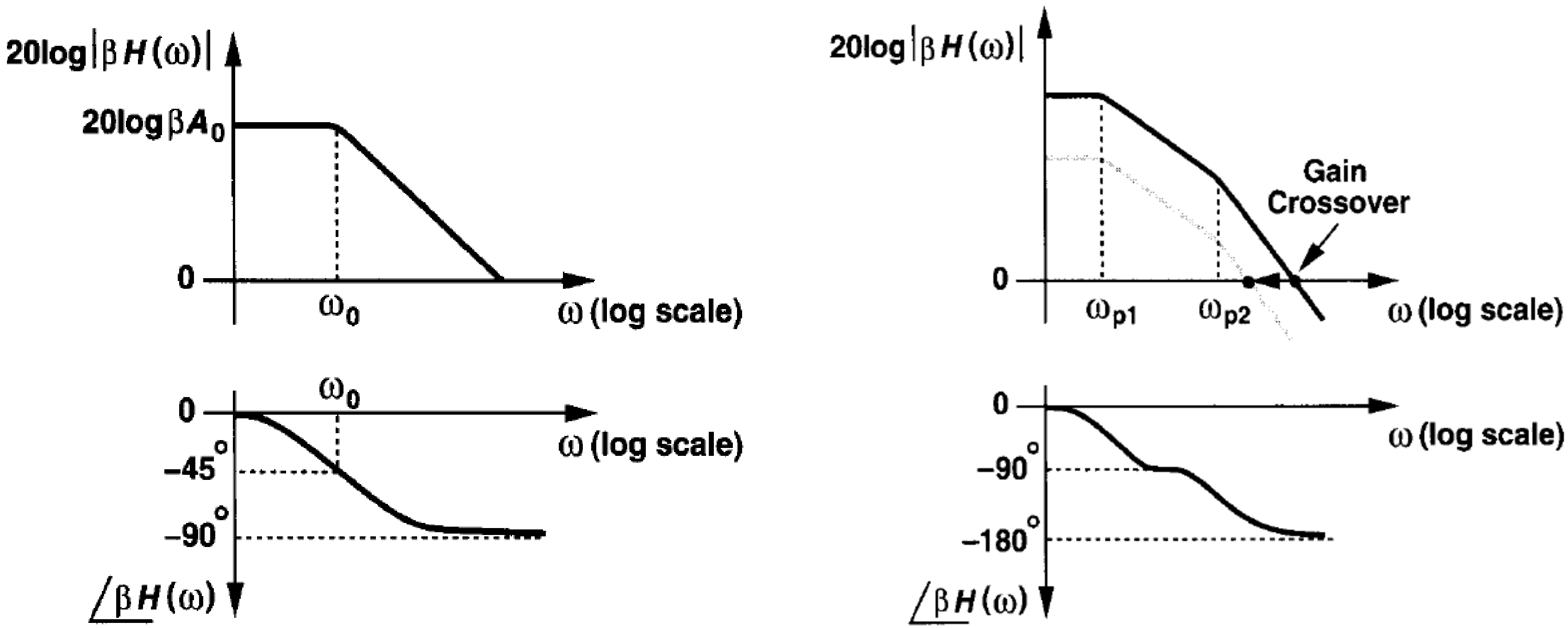
Stability of Feedback Systems

- Definition : Gain-crossover (GX) , Phase-crossover (PX)



Stability → GX earlier than PX (phase margin) , or
@ GX, phase shift less than -180 , or @PX Gain less than unity

Stability of Feedback Systems



Single Pole will not create phase shift greater than 90, hence always stable

2 Pole system will also be stable but with phase margin concern, Multiple Pole system will start to be instable

$\beta = 1$ is the worst condition, because, for $\beta < 1$, the GX moves leftwards, GX stays

Stability of Feedback Systems

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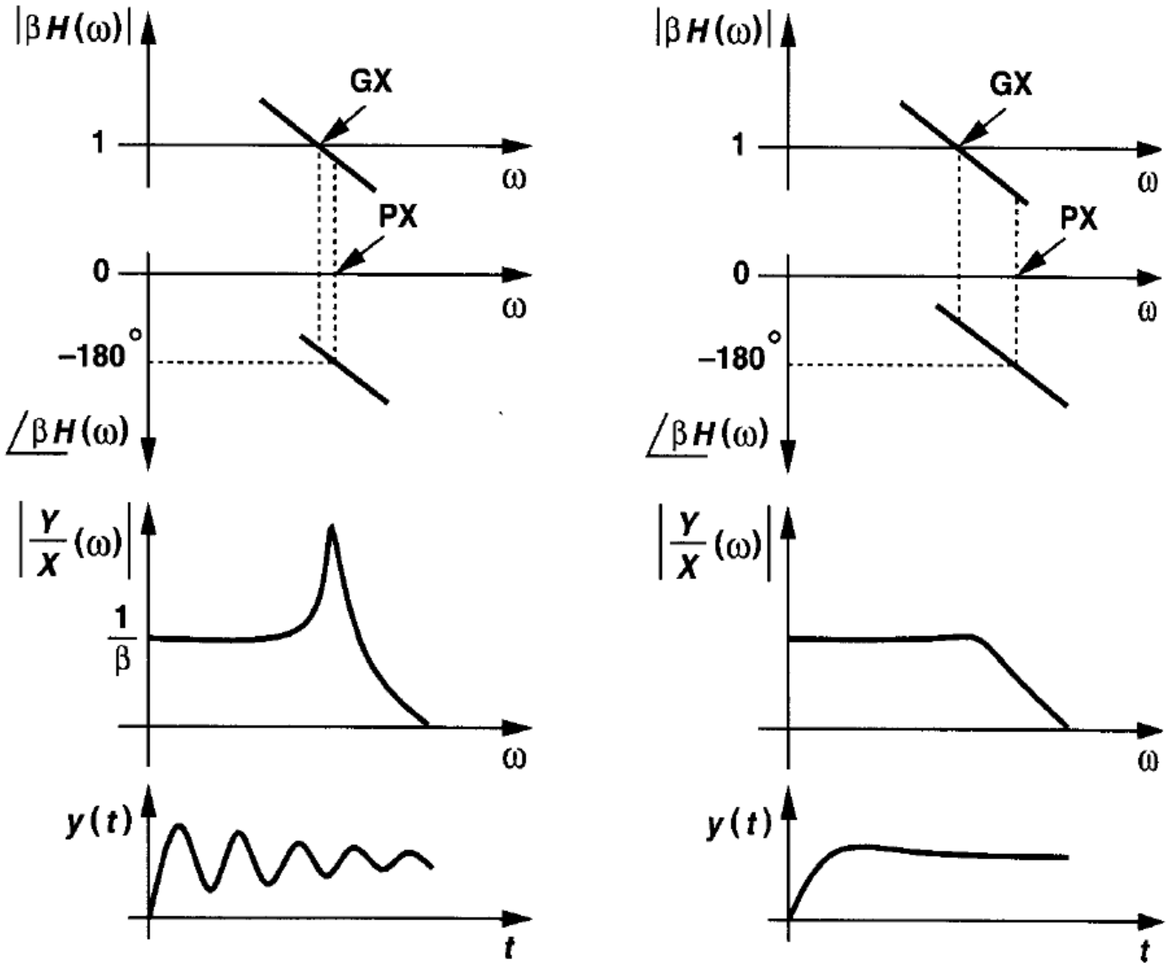
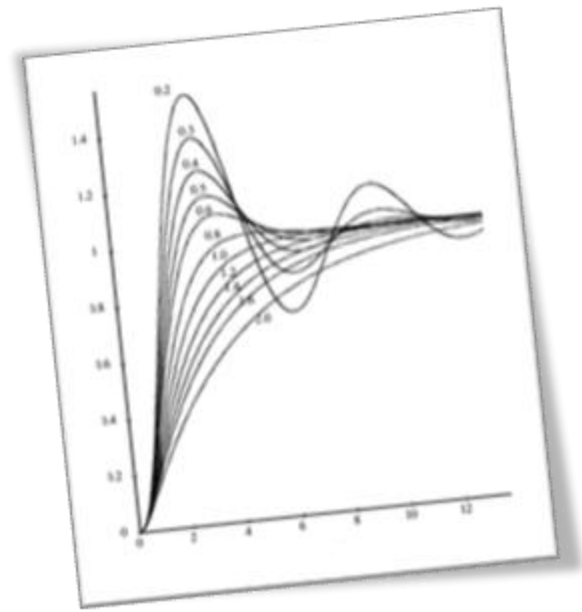
phase margin : how far away are GX ahead of PX



unit step response
of 2 pole systems



Stability of Feedback Systems

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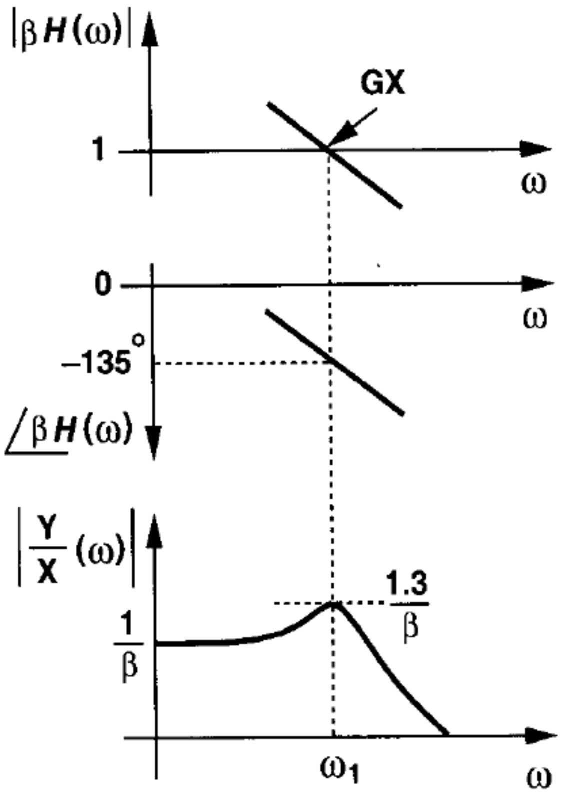
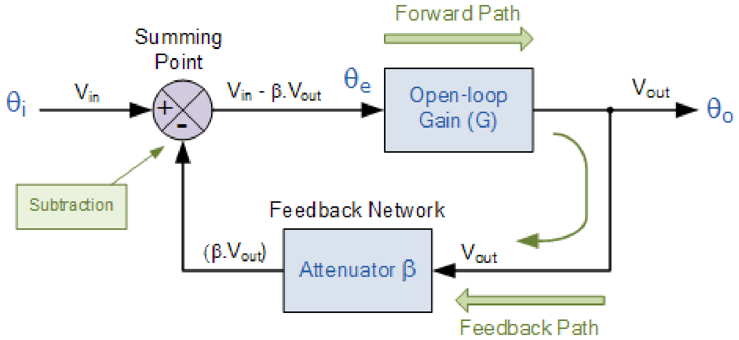
phase margin : how far away are GX ahead of PX

$$\frac{Y}{X}(j\omega) = \frac{H(j\omega)}{1+\beta H(j\omega)}$$



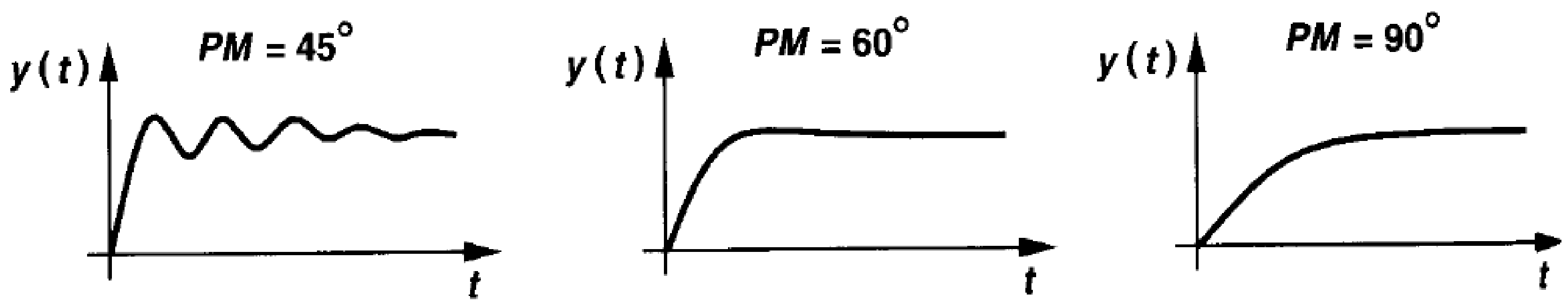
@ GX , βH(jω) =
1 · exp [-j (180° - PMargin)]

Stability of Feedback Systems



unit step response of feedback system, with different phase margin

usually set @ 60° for no over&undershoot
same as the 2 pole system transfer function
c.f. page 12



even though the 2 pole system is stable, it has the phase margin problem, the relative location of the first and second poles determines the phase margin … c.f. the example on next page

Stability of Feedback Systems

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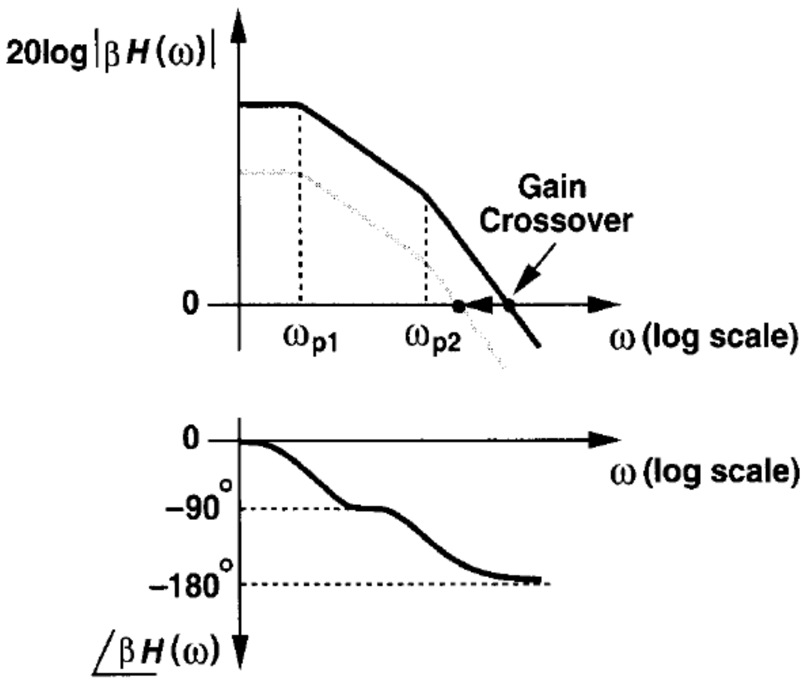
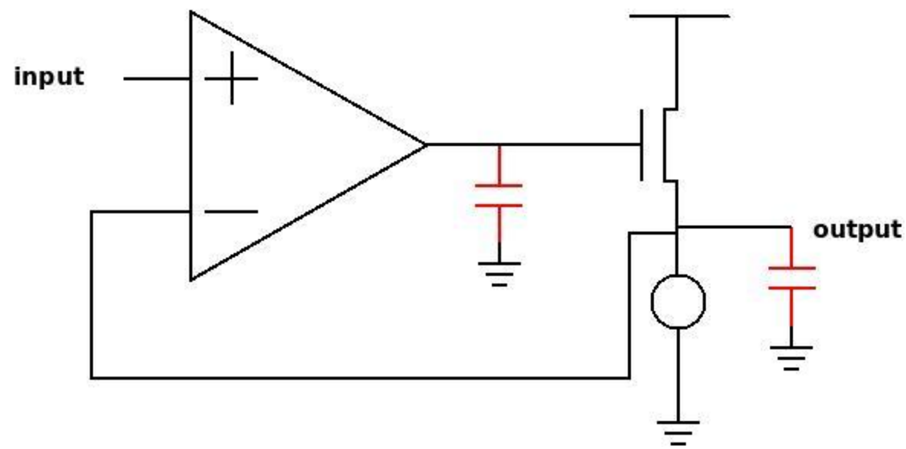
even though the 2 pole system is stable, it has the phase margin problem, the relative location of the first and second poles determines the phase margin





source follower is used to drive a large capacitive load, then the phase margin needs to be considered

however, all the analysis is based on small-signal large signal will also ring even if small signal has enough phase margin!!!

Compensation of 1 stage Amp

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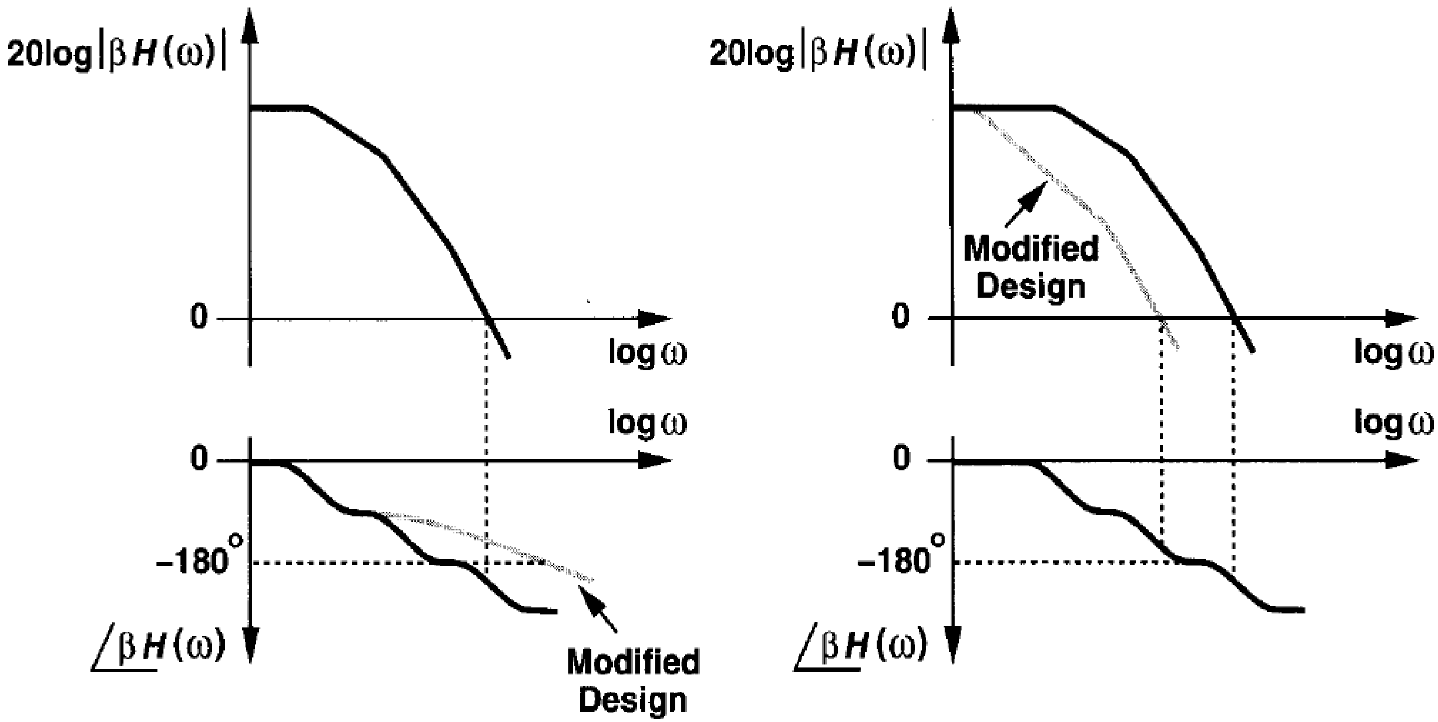
2 ways of compensating 1 stage Amp :

- ✓ reducing the amount of poles (less phase shift)
- ✓ moving dominant pole towards origin



Compensation of 1 stage Amp

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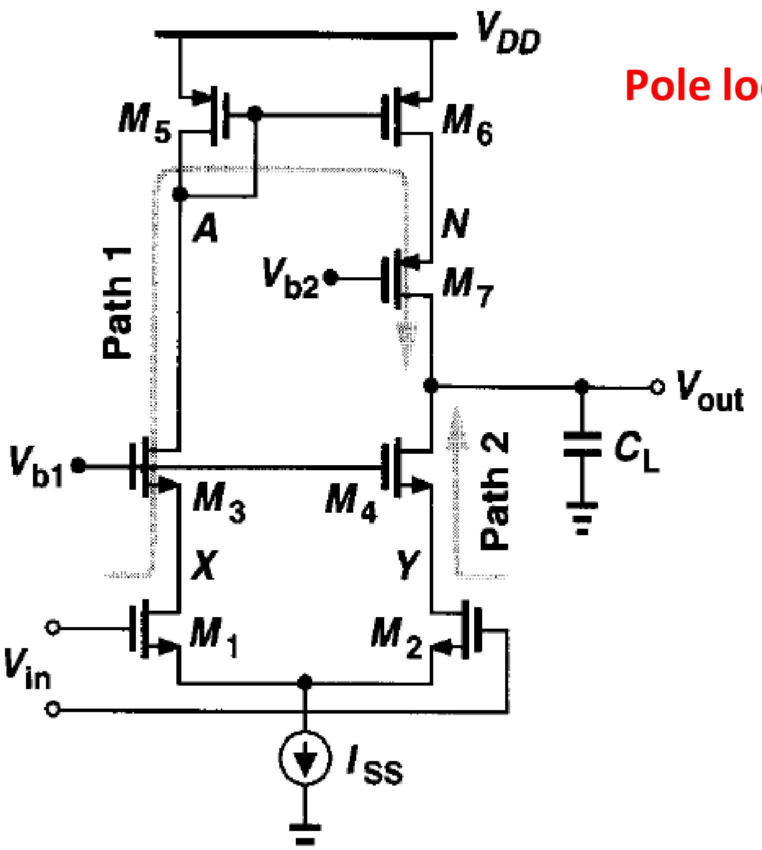
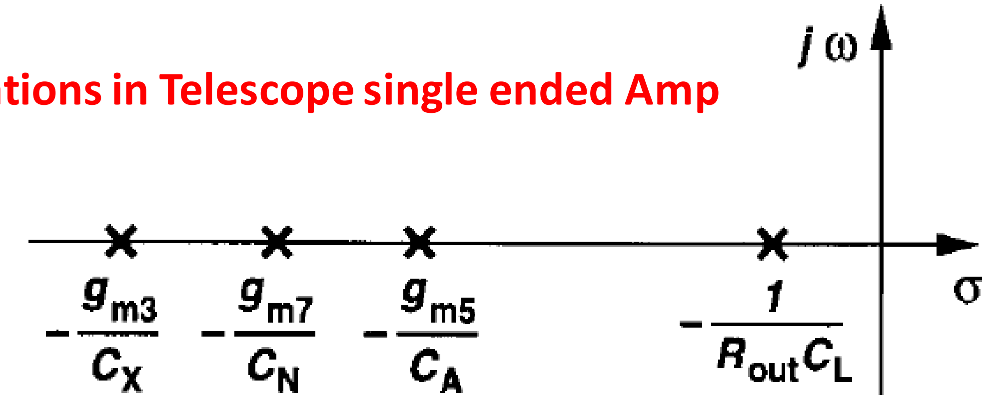
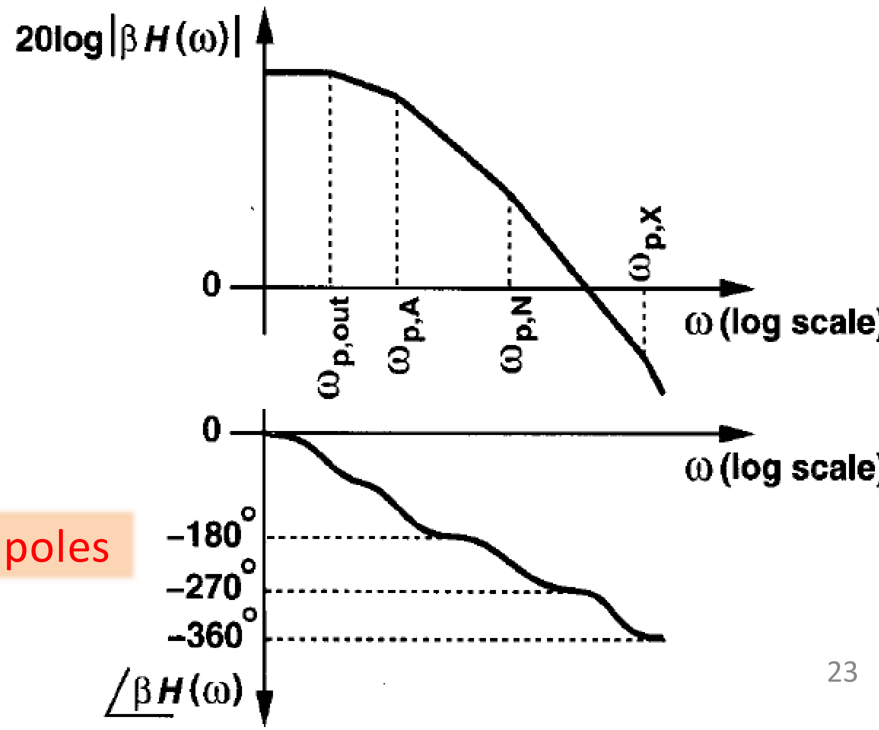


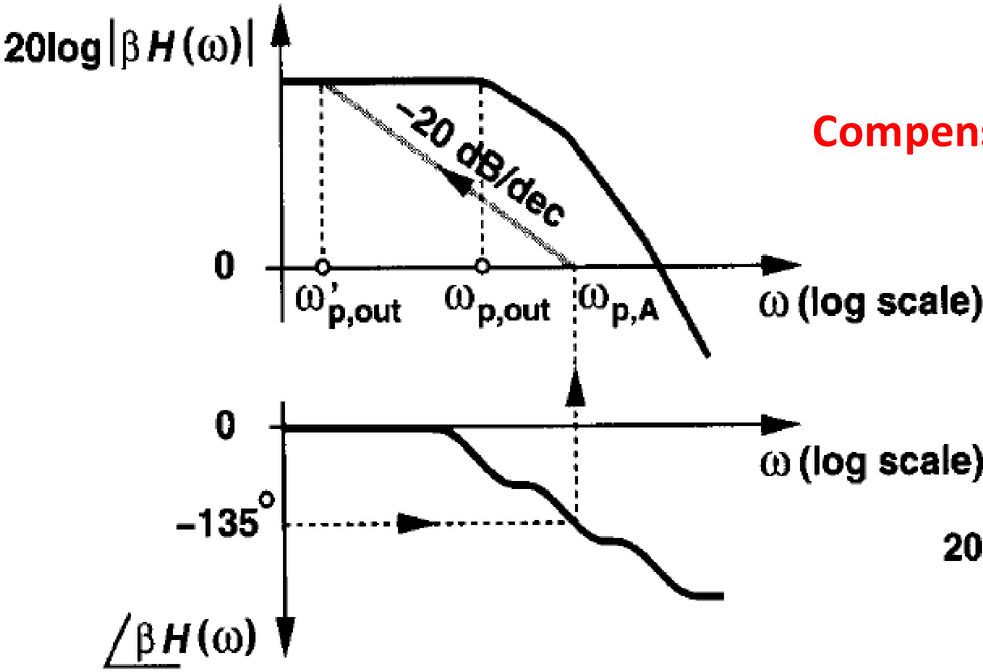
Pole locations in Telescope single ended Amp





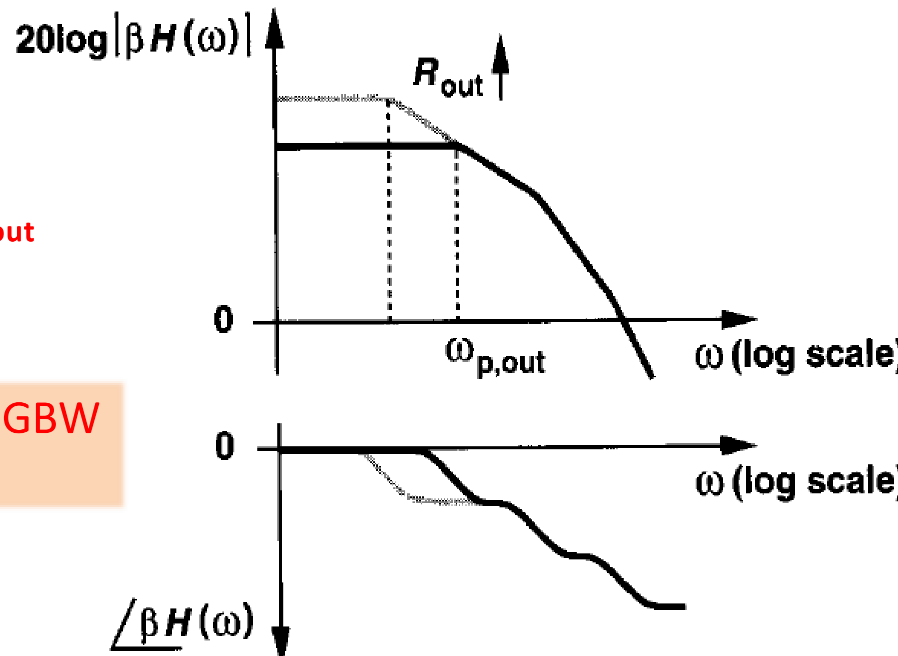
stability is a concern as for the non-dominant poles

Compensation of 1 stage Amp



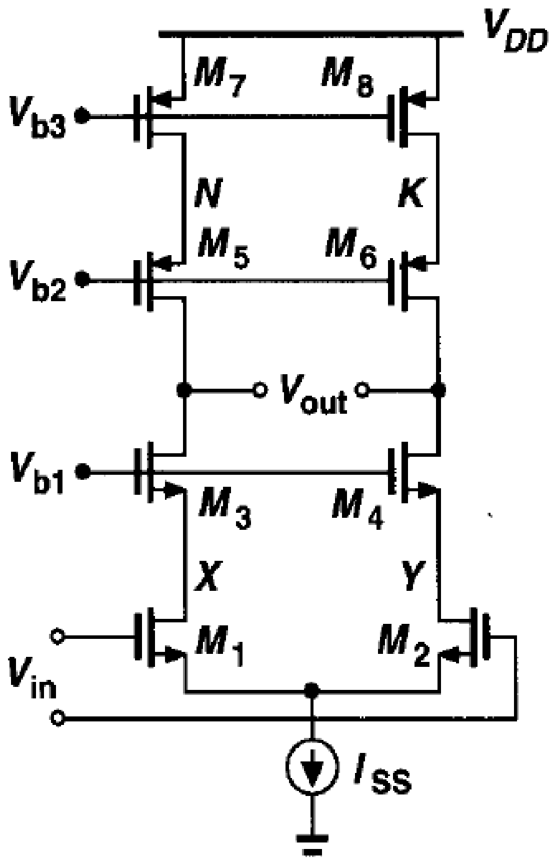
Compensating with larger C_{load}

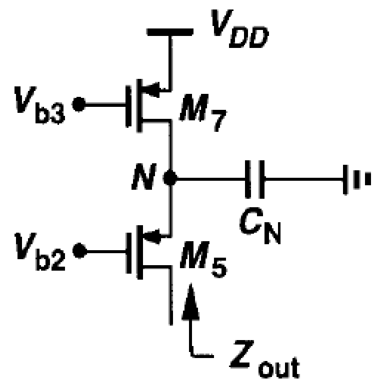
Compensation not valid with larger R_{out}

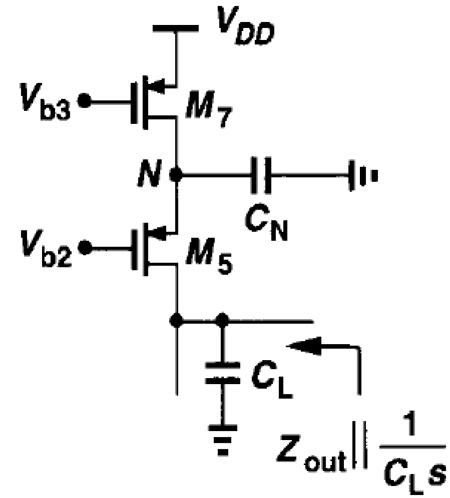


non-dominant poles need to be pushed above GBW depending on the phase margin required

Compensation of 1 stage Amp





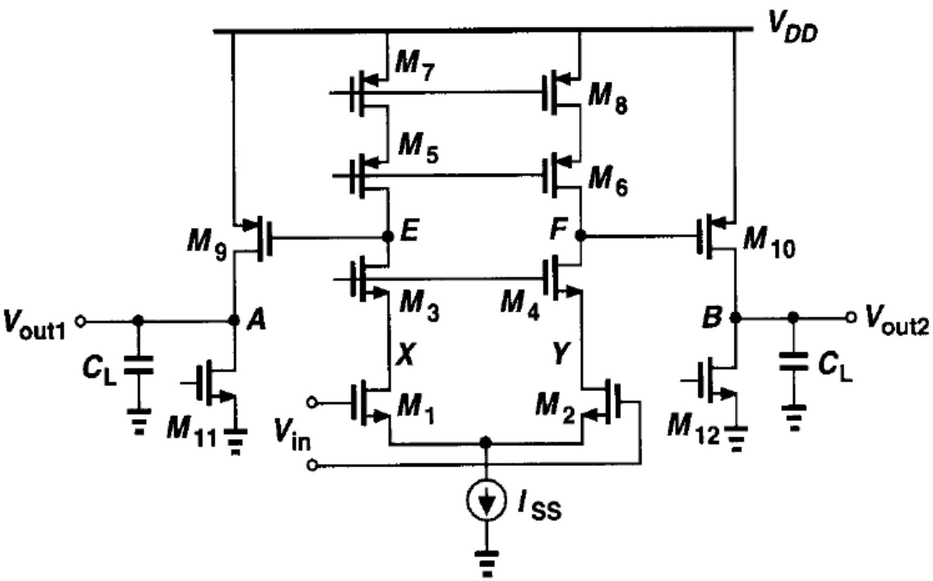


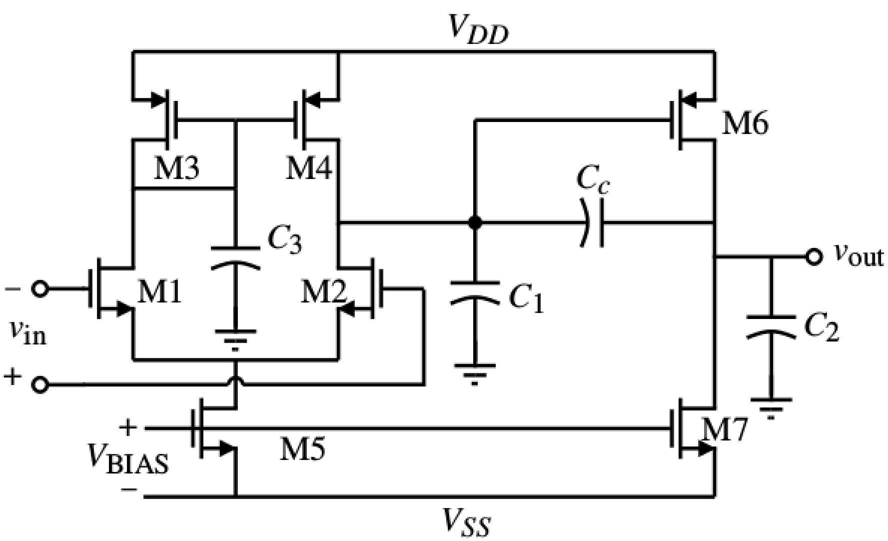
$$Z_{out} \| \frac{1}{C_L s} = \frac{(1+g_{m5}r_{O5})\dfrac{r_{O7}}{r_{O7}C_N s+1}\cdot\dfrac{1}{C_L s}}{(1+g_{m5}r_{O5})\dfrac{r_{O7}}{r_{O7}C_N s+1}+\dfrac{1}{C_L s}}$$

$$= \frac{(1+g_{m5}r_{O5})r_{O7}}{[(1+g_{m5}r_{O5})r_{O7}C_L + r_{O7}C_N]s+1}.$$

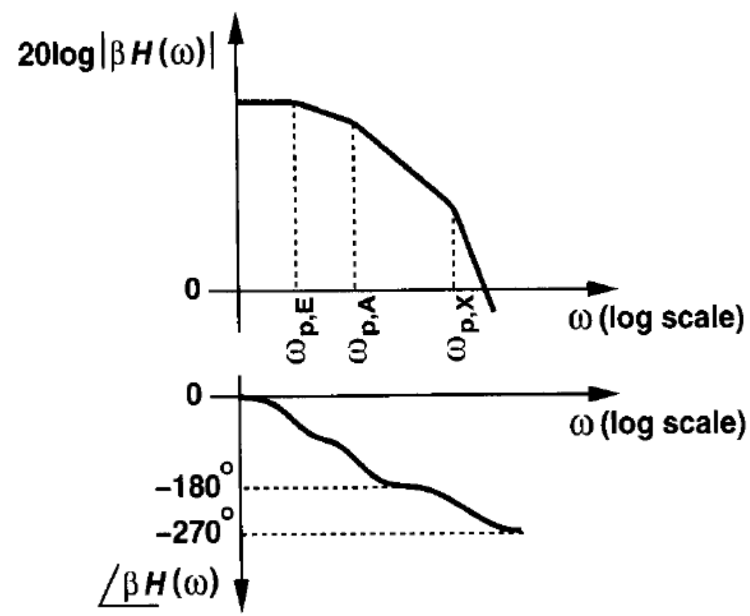
similar for fully-differential telescope structure but the poles at N and K are invisible or merged into the output pole

Compensation of 2 stage Amp





Pole locations in 2 stage Amps



Compensation of 2 stage Amp

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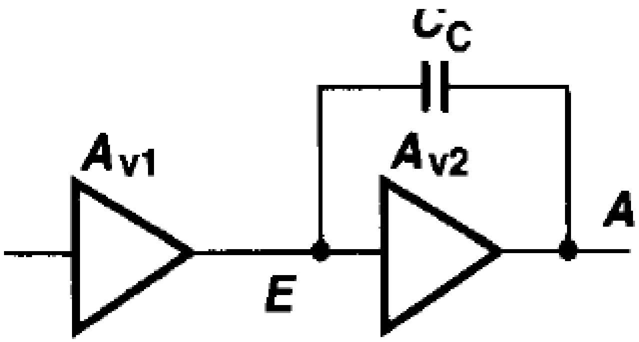
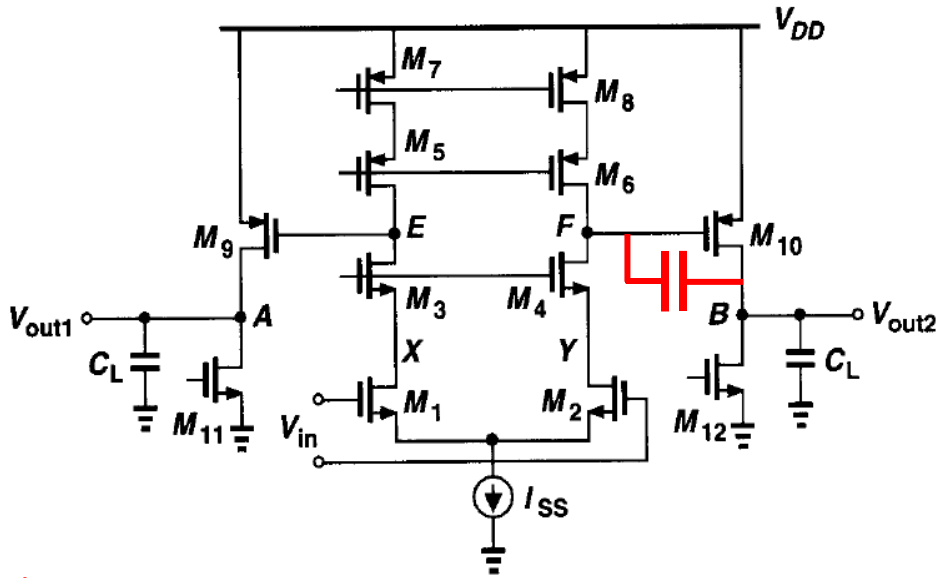
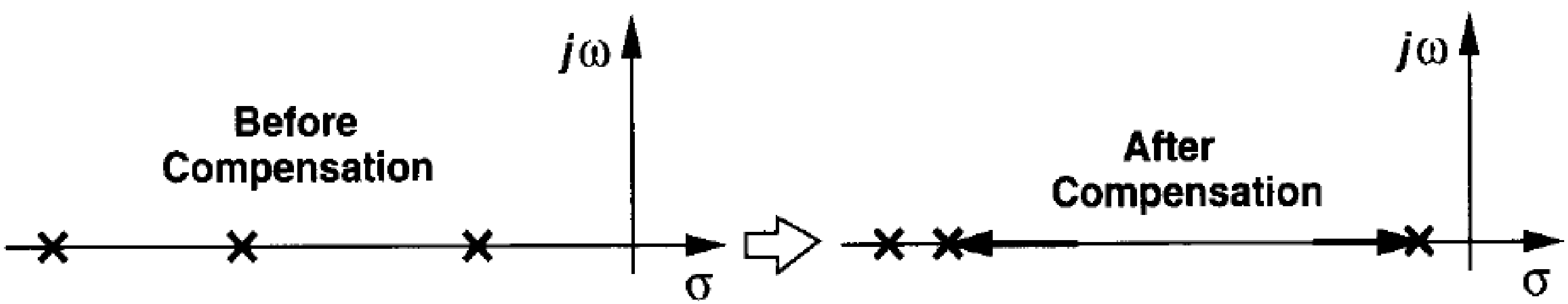
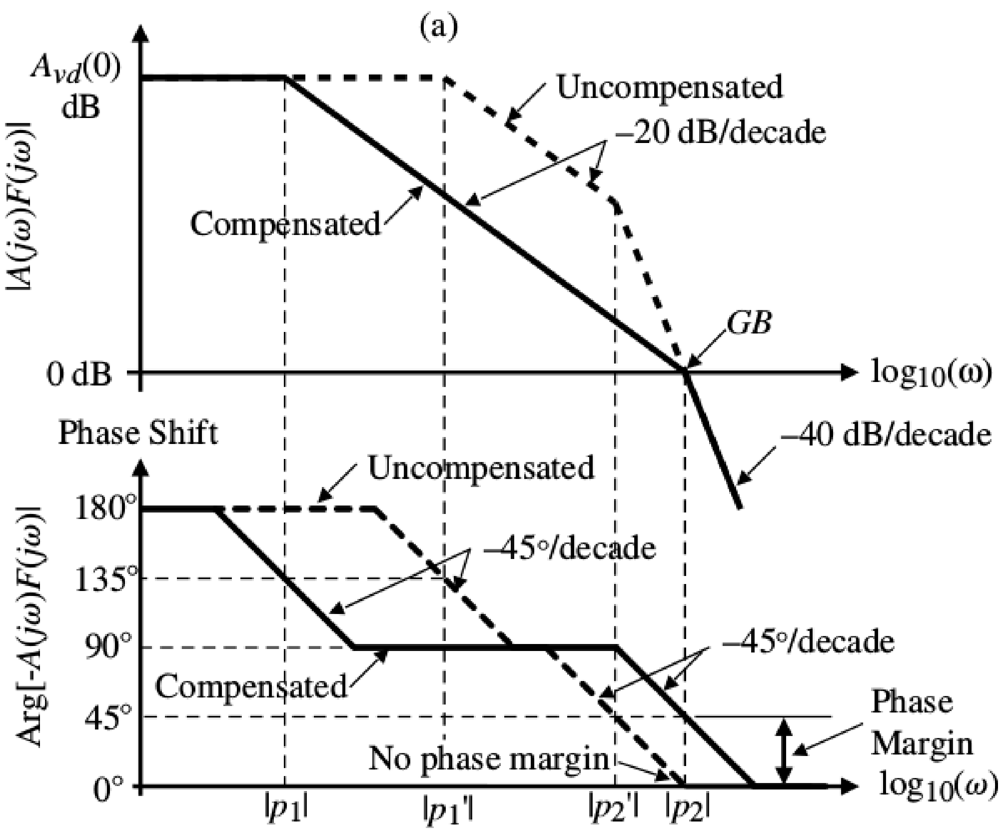




the effect of Miller Capacitor :
remember the Miller Effects from single CS stage



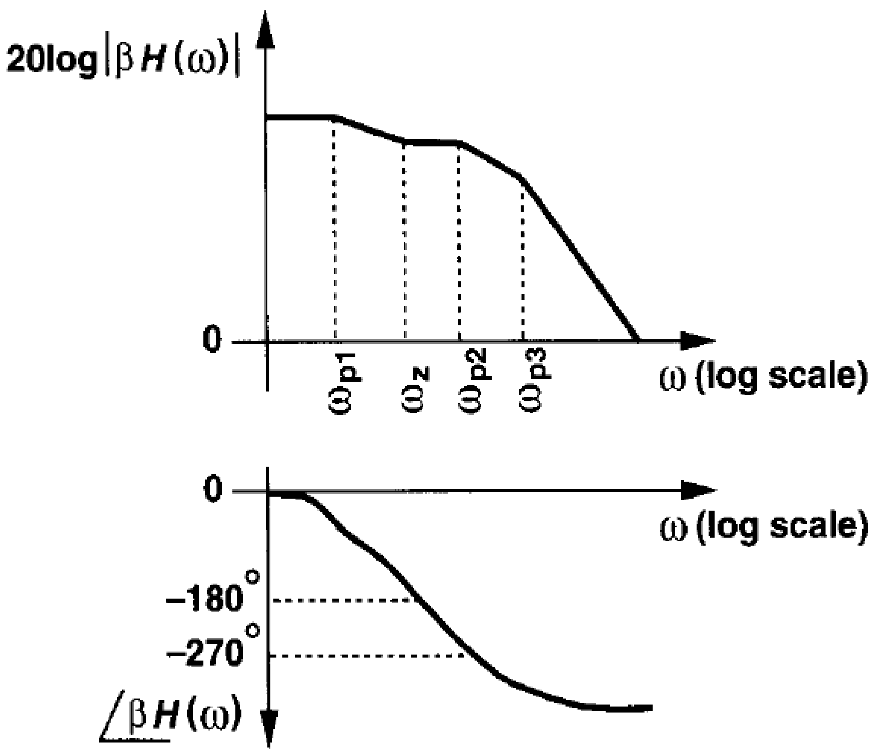
Compensation of 2 stage Amp



what we want to achieve with Miller Compensation

Ideal Case but with side effects

Compensation of 2 stage Amp



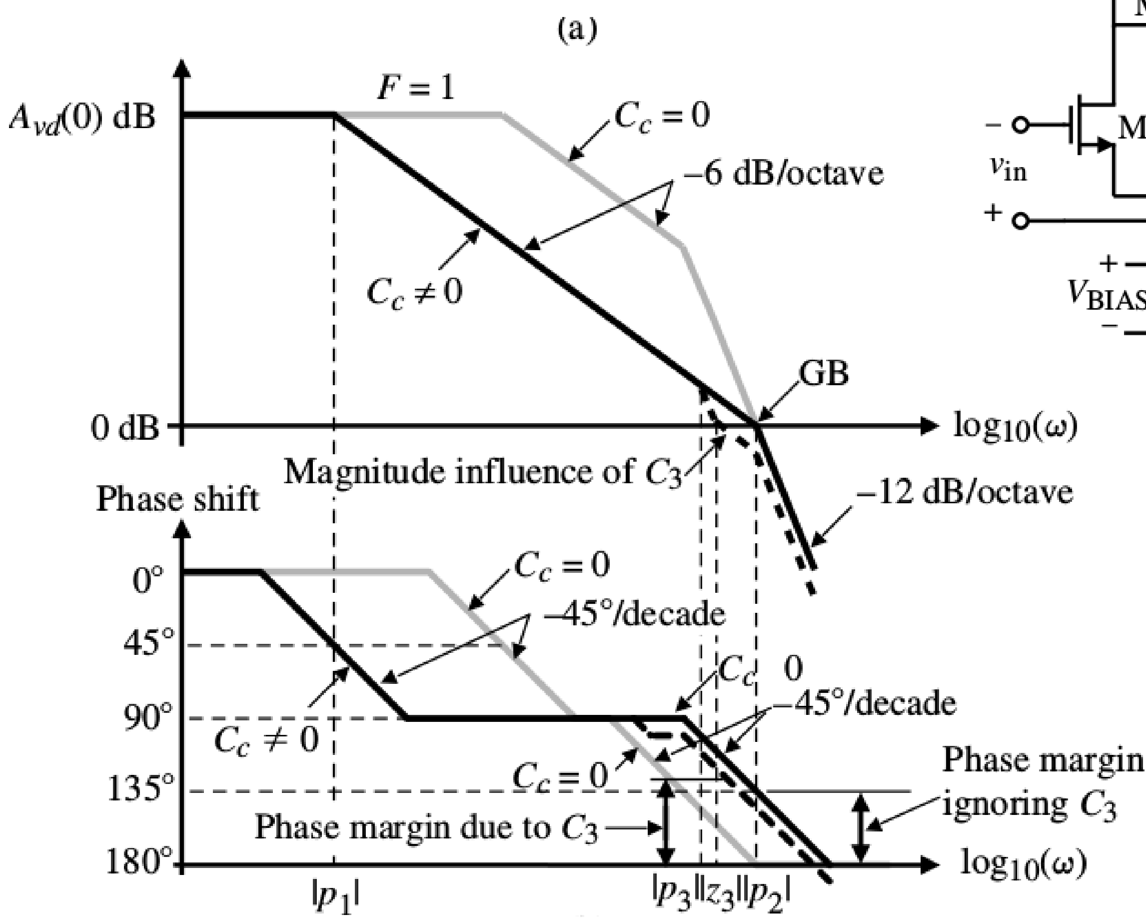
Effects of the RHP Zero, kills the phase margin, even though extends the bandwidth

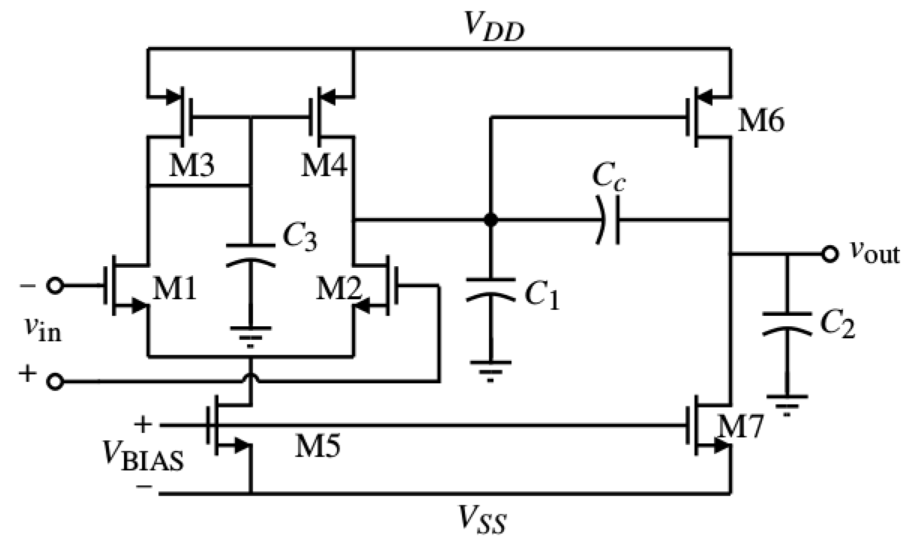
Or it has be placed carefully away from GBW

e.g. If RHP Zero placed at 10 times GBW, in order to achieve 60° phase margin

the second pole must be placed at least 2.2 times higher than GBW

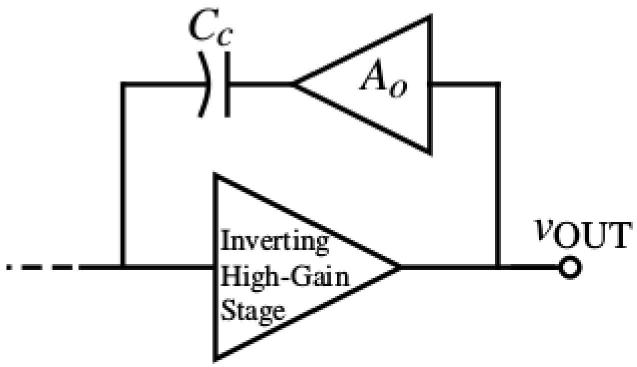
Compensation of 2 stage Amp





the effects of the Mirror Poles and zeros , marked at p_3 and z_3

Compensation of 2 stage Amp



how to remove the RHP zero,
feedback amplifier with $R_o = 0$

$$\frac{V_o(s)}{V_{in}(s)} = \frac{(g_{mI})(g_{mII})(R_I)(R_{II})}{1 + s[R_IC_I + R_{II}C_{II} + R_IC_c + g_{mII}R_IR_{II}C_c] + s^2[R_IR_{II}C_{II}(C_I + C_c)]}$$

$$p_1 \cong \frac{-1}{R_IC_I + R_{II}C_{II} + R_IC_c + g_{mII}R_IR_{II}C_c} \cong \frac{-1}{g_{mII}R_IR_{II}C_c}$$

$$p_2 \cong \frac{-g_{mII}C_c}{C_{II}(C_I + C_c)}$$

$$p_4 \cong \frac{-1}{R_o[C_IC_c/(C_I + C_c)]}$$

if R_o present

$$z_2 \cong \frac{-1}{R_oC_c}$$

Compensation of 2 stage Amp

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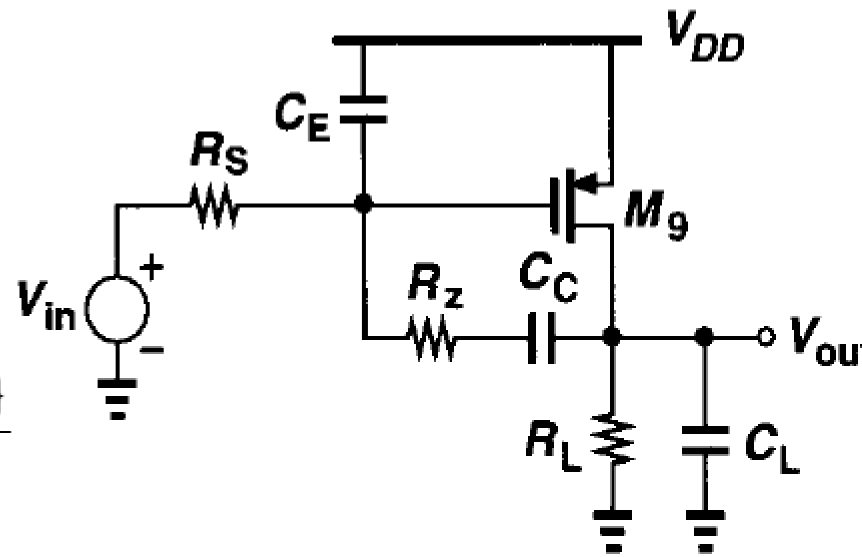
using nulling Resistance



$$\frac{V_{\text{out}}(s)}{V_{\text{in}}(s)} = \frac{a\{1 - s[(C_c/g_{mII}) - R_zC_c]\}}{1 + bs + cs^2 + ds^3}$$

with

$$a = g_{mI}g_{mII}R_IR_{II}$$

$$b = (C_{II} + C_c)R_{II} + (C_I + C_c)R_I + g_{mII}R_IR_{II}C_c + R_zC_c$$

$$c = [R_IR_{II}(C_IC_{II} + C_cC_I + C_cC_{II}) + R_zC_c(R_IC_I + R_{II}C_{II})]$$

$$d = R_IR_{II}R_zC_IC_{II}C_c$$

Compensation of 2 stage Amp

$$p_1 \cong \frac{-1}{(1 + g_{mII}R_{II})R_I C_c} \cong \frac{-1}{g_{mII}R_{II}R_I C_c}$$

$$p_2 \cong \frac{-g_{mII}C_c}{C_I C_{II} + C_c C_I + C_c C_{II}} \cong \frac{-g_{mII}}{C_{II}}$$

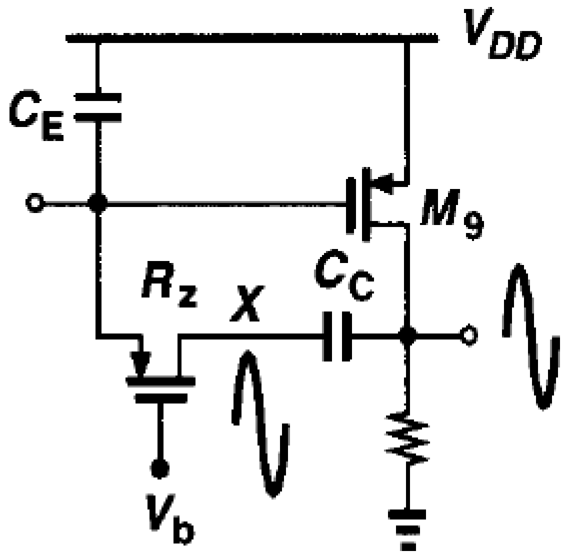
$$p_4 = \frac{-1}{R_z C_I}$$

$$z_1 = \frac{1}{C_c(1/g_{mII} - R_z)}$$

using nulling Resistance to cancel out the p_2, such that only the p_3 and p_4 remains

the bandwidth can be extended

Compensation of 2 stage Amp

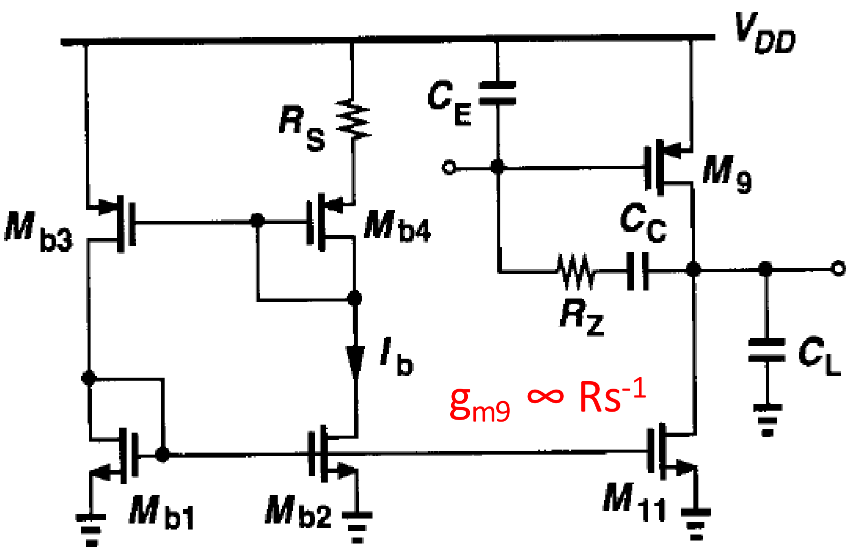


however problems :

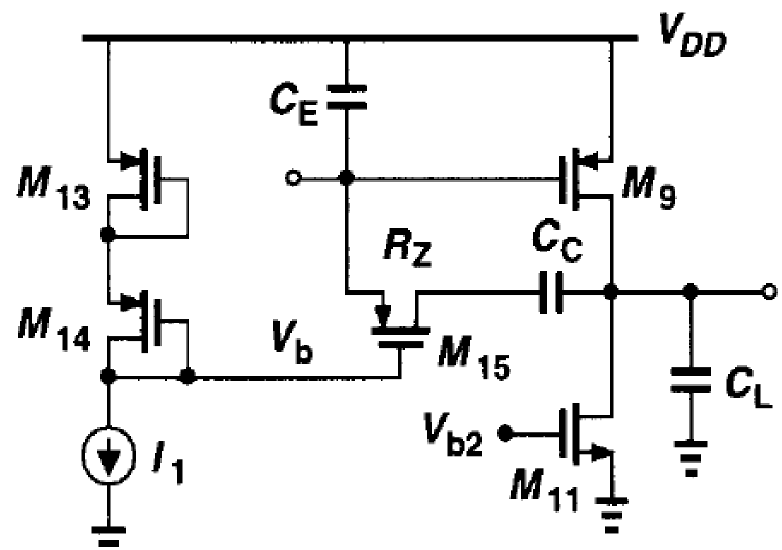
$$R_z = \frac{C_L + C_c}{g_{m9} C_c}$$

Depending on CL , needs to be flexible

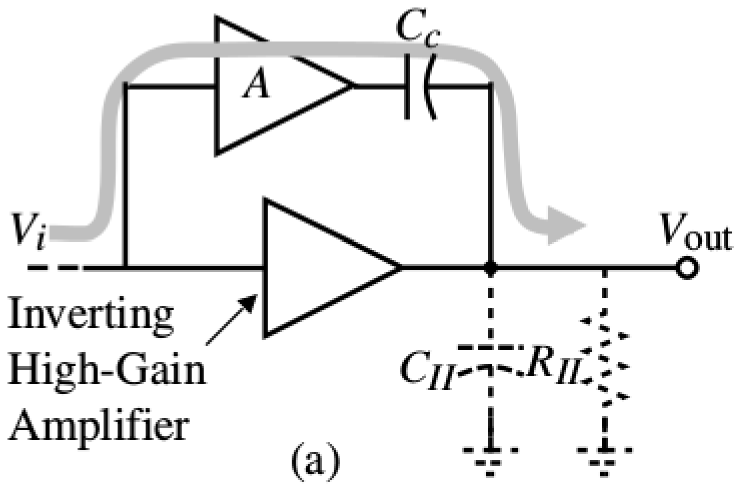
use the triode transistor to replace R_z,
but swing effect …… for fixed CL,
Vb anti-affect the process variation



$g_{m9} \propto Rs^{-1}$



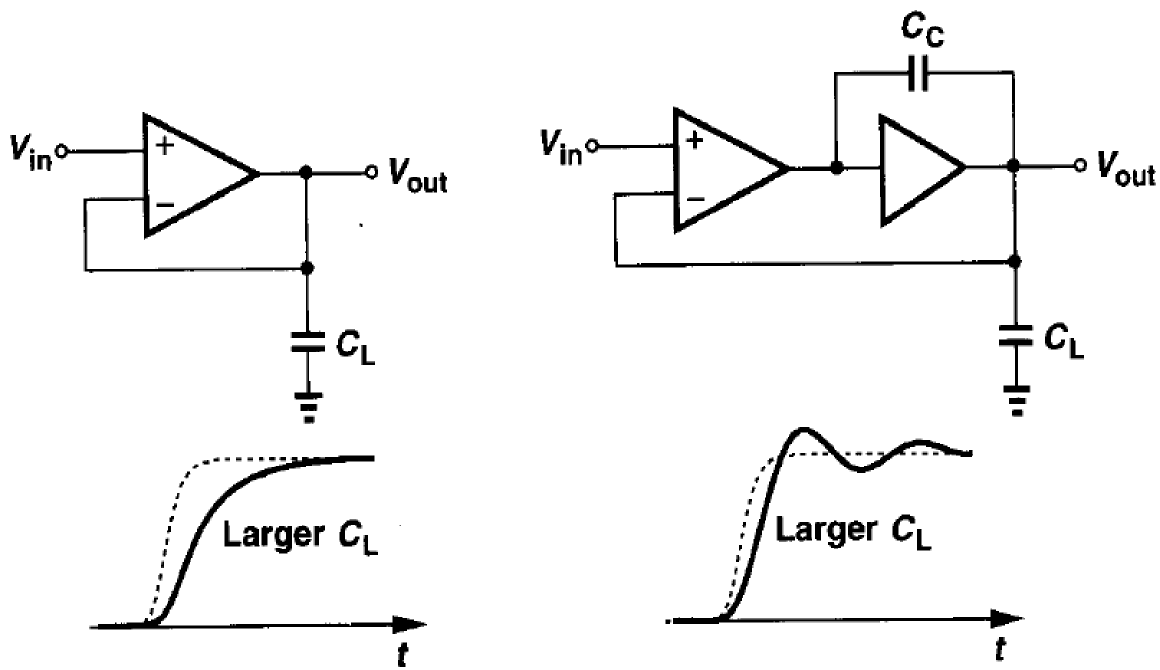
Compensation of 2 stage Amp



feedforward： moving the RHP zero to LHP zero

Compensation of 2 stage Amp

phase margin with different C_{load} and pole swapping!!!



what if the Loading Capacitor is extremely large,
then the poles are swapping
how does it look like ?